

ON THE RELATION BETWEEN
THE ALGEBRA OF CURRENTS AND
THE CLASSIFICATION GROUP FOR HADRONS

SUR LA RELATION ENTRE
L'ALGEBRE DES COURANTS ET
LE GROUPE DE CLASSIFICATION DES HADRONS

Thèse

présentée à la Faculté des sciences de l'Université de Genève
pour obtenir le grade de docteur ès sciences physiques

par

Carlos A. SAVOY

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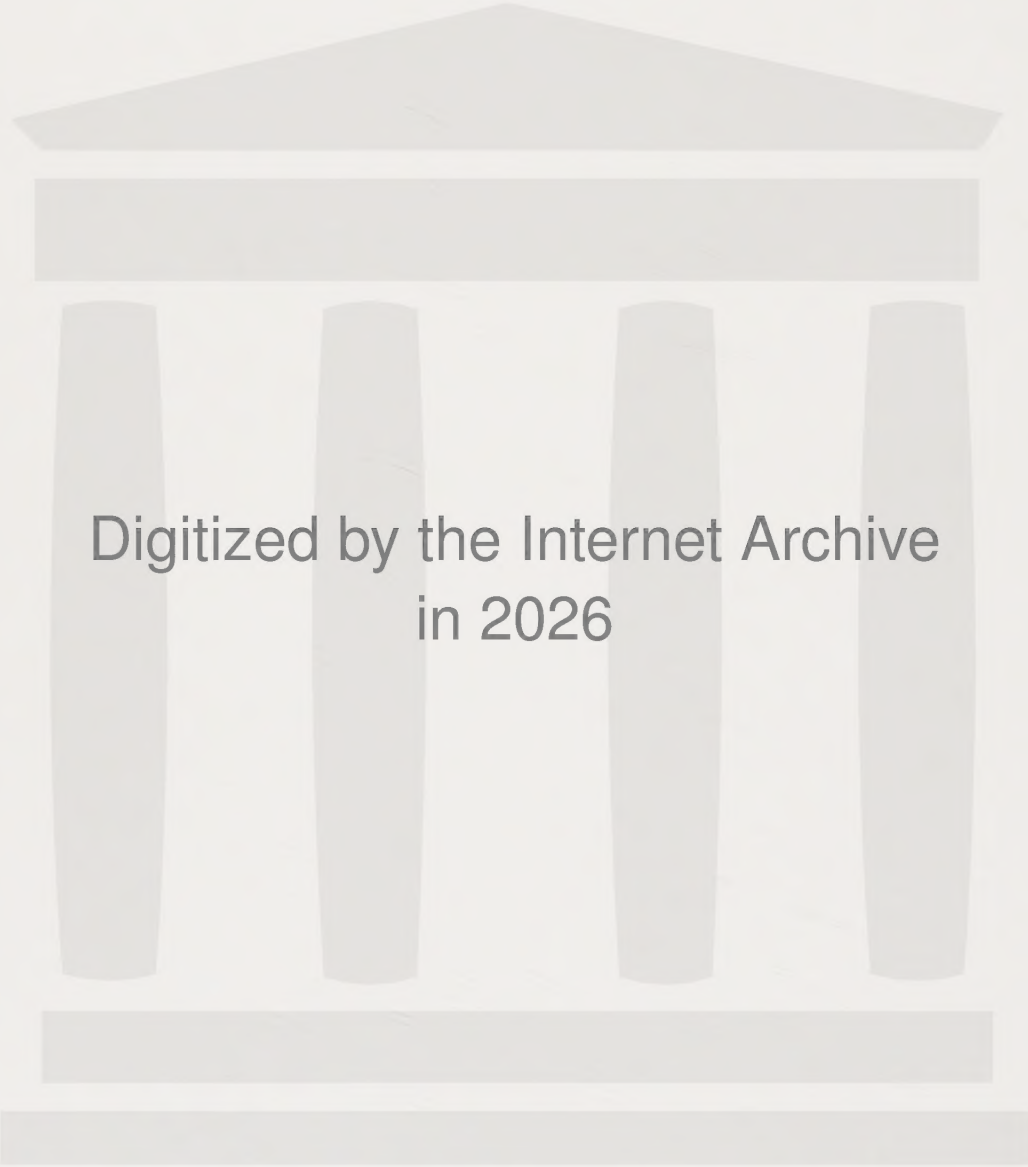
SAO PAULO (Bresil)

THESE No 1623

GENEVE

Imprimerie de la Section de Physique

1973



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Genève, le 17 juillet 1973

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Thèse No 1623



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INTRODUCTION

The proliferation of hadronic resonances compelled particle physicists to arrange them in some systematic sequences. The conserved quantum numbers soon appeared to be insufficient and the approximate internal symmetry group, $SU(3)$, had also to be enlarged for this purpose. At present, the most successful group for the classification of hadrons is certainly $SU(6) \times O(3)$ ¹⁾. Very recently, some evidence has appeared in favour of the symmetric oscillator quark model¹⁾.

[The classification of mesons and baryons inside the $SU(6) \times O(3)$ multiplets is discussed in Chapter I.]

The postulated algebra for the weak and electromagnetic charges²⁾ gives much information on the structure of hadrons as it should be saturated, i.e., realized by hadronic states. Because the algebra of these vector and axial charges is not an approximate symmetry of Nature, its saturation is quite involved. There are some advantages which justify the use of a $p \rightarrow \infty$ reference frame.

[The charge algebra and its saturation are the subject of Chapter II.]

However, if the algebra of charges is the same as a subalgebra of the hadron classification group [as for $SU(6) \times O(3)$], they could be related by a unitary operator which would then be responsible for the mixing of many different multiplets of the classification group occurring in the matrix elements of the charges.

Some time ago we proposed a simple "mixing operator" along these lines, on an empirical basis³⁾. It was quite successful phenomenologically and gave many good predictions for decays of mesons and baryons^{3,4)}. Even the meson masses^{4,5)} and the baryon magnetic moment⁶⁾ have been studied in this way. An attempt to apply the mixing operator to the production of resonances has been made⁷⁾. This notwithstanding, the model was not completely satisfactory: the calculations were done in a perturbative way, and, on the other hand, a physical interpretation was missing in our scheme.

Recently, Melosh⁸⁾ has studied the relation between the "collinear" symmetry group and the algebra of the "good" charges in the free-quark model. He has shown that the charges differ from the generators of the symmetry group by a unitary transformation.

In this work⁹⁾, we construct such a mixing operator for meson and baryon states on the basis of the Melosh transformation, and the resulting scheme looks much like the one proposed in Ref. 3, thus providing a physical interpretation for the latter. But it has some advantages, as it is much more explicit: calculations can be done exactly, in a non-perturbative way, and, in this context, the operators entering into the model have the same simple algebraic structure as in the Melosh work on the free-quark model.

Some comments are in order here. The usual way to study the phenomenological consequences of symmetry breaking is what could be called the "extrapolative approach": one looks for the algebraic properties of physical observables in the free-quark field model and then postulates the same structure for the corresponding hadronic operators. The current algebra has been derived in this way by Gell-Mann, and it has been successfully tested. Also the Gell-Mann/Okubo mass formula and the U-spin singlet character of the photon can be inferred from the free-quark model and are consistent with experiment. But we do not expect this extrapolative approach to work everywhere for a trivial reason: quarks are usually classified in the fundamental representation of the relevant symmetry group, so that operators must have simple symmetry properties in the free-quark model as they are taken between two one-quark states; instead, hadrons are classified in higher representations, which then allow for more complicated pieces of those operators to contribute too. So, it is a conspicuous fact that this method works in many cases, but there must be a reason for this and we have to be careful as this approach is not very justified from a theoretical point of view.

We must honestly say that some of the more interesting results that will be derived here can also be obtained in this "extrapolative approach"¹⁰⁾ [but, indeed, they were given earlier in Ref. 3.]. However, we should try to understand, at least intuitively, where these group theoretical properties

come from, in order to improve and to enrich the model towards a theory. In this sense, an explicit model like that presented in our work^{9,11)} could be useful.

These comments were needed in order to motivate the treatment given here to the problem of the saturation of the chiral algebra of charges on the basis of the SU(6) classification of elementary (?) particles. Starting from a simple model of hadrons as made of quarks, we use the Melosh transformation⁸⁾ to obtain mixing operators for mesons and baryons; the charge operators are then explicitly given. For simplicity, the breaking of SU(3) is not taken into account⁶⁾.

[The problem of mixing, the Melosh transformation, and the main hypotheses in our approach are given in Chapter III.]

For mesons, the charge operators turn out to be uniquely determined; they have the good properties under charge conjugation and a very simple algebraic structure⁹⁾. By using PCAC one can calculate the pionic couplings which result from the axial charge matrix elements and they compare very well with experiment. In order to estimate the dependence of the matrix elements on the spatial configurations of the quarks inside the mesons, we consider a harmonic oscillator model for the spectrum, which gives reasonable results for the low-lying mesons⁹⁾. An application of the mixing operator to the electromagnetic decays of mesons is also made⁹⁾.

[The study of the mixing operator in the case of mesons is presented in Chapter IV.]

For baryons, some ambiguities in the charge operators remain⁹⁾ which are discussed here, and they can be avoided once we give the spectrum of states; the charges are consistent with definite statistics for quarks inside baryon states. Then we symmetrize these states and determine, in terms of their spatial configuration, the axial charge matrix elements, which agree fairly well with the experimental data¹¹⁾. By assuming the spectrum of the harmonic oscillator quark model¹⁾ we can calculate the spatial dependence of the matrix elements, and we find reasonable values for the low-lying baryons; for the higher states some modifications are needed¹¹⁾. In this model many of the states which have not yet been seen,

or have been discovered only recently, turn out to be less coupled to nucleons than the better known resonances¹¹⁾. The Johnson-Treiman relations¹²⁾ are derived from the contribution of the resonances in the s-channel¹¹⁾. The meson-baryon t-channel amplitudes which result from the calculated coupling constants have the structure proposed by Cabibbo, Horwitz and Ne'eman¹³⁾, for asymptotic energies, but is is approximately valid even at intermediate energies¹¹⁾.

[The application of the mixing operator to baryons is made in the last Chapter.]

CHAPTER I

SU(6): A CLASSIFICATION GROUP FOR HADRONS

In this chapter we review the good evidence for SU(6) as a classification group for hadrons. For briefness, we renounce to take a historical approach and look directly into the justifiability of the predilection that particle physicists have for this group, and also to its systematics. So, we separate the ideas of SU(6) as a symmetry and as a classification group, although, traditionally, these concepts have been closely related: the SU(6) symmetry is badly broken, and we could hardly infer the classification group from it. We should rather look for qualitative regularities inside the several dozens of resonances in the "Particle Data Group" Table¹⁴⁾, in order to group them into the multiplets of SU(6). This has been done by many people in the last decade and our belief in SU(6) as a classification group comes from this patient work¹⁾.

The group SU(6) has 35 generators which we denote by Q_μ^α ($\alpha = 0, 1, 2, \dots, 8$; $\mu = 0, 1, 2, 3$; $Q_0^0 = I$ excluded), that reads in its 6-dimensional fundamental representation (indicated by the symbol $\sim$), $Q_\mu^\alpha \sim \frac{1}{2} \lambda^\alpha \sigma_\mu$. Here, λ^α are the eight matrices of the 3×3 representation of SU(3), σ_i ($i = 1, 2, 3$) are the Pauli matrices, $\sigma_0 = I$ and $\lambda^0 = I$. A physically interesting subgroup is the SU(3) $\times$ SU(2) generated by $Q_0^\alpha \equiv Q^\alpha$ and $Q_i^0 \equiv S_i$.

I.1 CLASSIFICATION OF QUARKS

We can now introduce the quark idea (either as mathematical or as physical objects). We associate with the fundamental representation (6) of SU(6), six quark states (q) which we denote by $p^\uparrow, p^\downarrow, n^\uparrow, n^\downarrow, \lambda^\uparrow, \lambda^\downarrow$. The operators $Q^\alpha \sim \frac{1}{2} \lambda^\alpha$ are now identified with the generators of SU(3), $Q^i = I^i$ ($i = 1, 2, 3$) being the isospin and $(2/\sqrt{3}) Q^8 = Y$ the hypercharge operators; quarks are then of the type $p(I = \frac{1}{2}, I_3 = \frac{1}{2}, Y = \frac{1}{3})$, $n(I = \frac{1}{2}, I_3 = -\frac{1}{2}, Y = \frac{1}{3})$ or $\lambda(I = 0, Y_3 = -\frac{2}{3})$. The $\bar{6}$ representation is associated with antiquarks ($\bar{q}$).

I.2 CLASSIFICATION OF MESONS

The bosonic resonances can all be classified in octets and singlets of SU(3); there is no clear evidence against this rule. As in SU(3), $\underline{3} \times \underline{\bar{3}} = \underline{1} + \underline{8}$ mesons can be considered as pairs of SU(3) quarks (p, n, λ) and antiquarks $(\bar{p}, \bar{n}, \bar{\lambda})$. We jump to SU(6) by giving spin- $\frac{1}{2}$ to these quarks. Now $\underline{6} \times \underline{\bar{6}} = \underline{1} + \underline{35}$, so that mesons are to be classified in singlets and 35-plets of SU(6). In the subgroup SU(3) $\times$ SU(2) defined above, the singlet is an SU(3)-singlet, S = 0 state and the 35-plet decomposes in an SU(3)-octet with S = 0 and a SU(3)-octet and an SU(3)-singlet, both with S = 1. As the spin of the known meson resonances can be 2, 3, and even more, an "orbital" angular momentum $\vec{L}$ is introduced and the meson spin is defined as $\vec{J} = \vec{L} + \vec{S}$. In other words, we enlarge our classification group to SU(6) $\times$ O(3). This $\vec{L}$ can eventually be interpreted as the orbital angular momentum of the $q\bar{q}$ pair.

On phenomenological grounds¹⁾ we can easily identify the $\underline{35} + \underline{1}$ states with L = 0, which we group as follows:

$$\underline{1}, S=0: \eta' \quad \underline{8}, S=0: \pi^{\pm}, K^{\pm}, \bar{K}^{\pm}, \eta \quad \underline{8} + \underline{1}, S=1: \rho^{\pm}, K^{*\pm}, \bar{K}^{*\pm}, \omega, \phi.$$

In a $\underline{35} + \underline{1}$ with L = 1, J can be 0, 1 or 2. Some states are missing (and are replaced by a "?"), others are just welcome candidates to resonances and are quoted inside brackets []:

$$\underline{1} + \underline{8}, S=0, L=1: B^{\pm}, Q_B^{\pm}, \bar{Q}_B^{\pm}, ?, ?$$

$$\underline{1} + \underline{8}, S=1, L=1: (J=2) A_2^{\pm}, K^{**\pm}, \bar{K}^{**\pm}, f, f'$$

$$(J=1) [A_1^{\pm}], Q_A^{\pm}, \bar{Q}_A^{\pm}, D, [E]$$

$$(J=0) \delta^{\pm}, [\chi^{\pm}, \bar{\chi}^{\pm}], [\epsilon], [S].$$

Masses and other properties of the above-mentioned resonances can be found in the "Particle Data Group" Table¹⁴⁾. For L $\geq$ 2 the situation is less clear and it is discussed in recent review articles¹⁾.

In Appendix I we give an explicit realization of the meson states in terms of SU(6) $\times$ O(3) $q\bar{q}$ states.

I.3 CLASSIFICATION OF BARYONS

Experimentally, all the baryon resonances can be classified in SU(3)-singlets, -octets and -decuplets (although many strange states are still missing); these are the representations which can be constructed from three SU(3)-triplets: $\underline{3} \times \underline{3} \times \underline{3} = \underline{1} + \underline{8} + \underline{8} + \underline{10}$. Therefore, we can associate baryons to three-quark states, qqq, which, in passing to SU(6) result in states belonging to the representations 56, 70, 20, as $\underline{6} \times \underline{6} \times \underline{6} = \underline{56} + \underline{70} + \underline{70} + \underline{20}$. In terms of the subgroup SU(3) $\times$ SU(2) they decompose as follows:

$$\underline{56}: \underline{8} \text{ S} = \frac{1}{2}, \underline{10} \text{ S} = \frac{3}{2}$$

$$\underline{70}: \underline{8} \text{ S} = \frac{3}{2}, \underline{8} \text{ S} = \frac{1}{2}, \underline{10} \text{ S} = \frac{1}{2}, \underline{1} \text{ S} = \frac{1}{2}$$

$$\underline{20}: \underline{8} \text{ S} = \frac{1}{2}, \underline{1} \text{ S} = \frac{3}{2} .$$

Because baryon resonances have been detected which have spin $\frac{5}{2}$, $\frac{7}{2}$, $\frac{9}{2}$, and higher, we introduce also here an orbital angular momentum $\vec{L}$ (eventually interpreted as the relative angular momentum of the quark system), the spin of the resonances being $\vec{J} = \vec{L} + \vec{S}$, in this SU(6) $\times$ O(3) classification group.

The SU(3) octet of stable baryons (N, Σ , Λ , Ξ) and the well-known resonances of the $J = \frac{3}{2}$ decuplet (Δ , Y^* , Ξ^*) are usually assigned to a 56, $L = 0$. There is good evidence for a 70 $L = 1$, a 56 $L = 2$ and the so-called Ropper resonance which has been classified in either another 56 $L = 0$, or a 20 $L = 1$ (presently the first assignment is favoured). Some recently discovered resonances have been associated with a possible 70 $L = 2$ and a 20 $L = 1$. The proliferation of states for $L \geq 1$ is so high that a discussion of the experimental evidence for an SU(6) classification becomes very involved and we refer to the literature on the subject¹⁾. Our guess for the classification of some non-strange resonances can be inferred from the tables in Chapter V.

Many representative states in the 56, 70 and 20 representations of SU(6), together with our notation for them, are given in Appendix II.

I.4 THE SYMMETRIC MODEL

Up to now, we have not attached any physical meaning to quarks. But the easiest way to understand this appealing quark structure of hadrons would be by supposing that quarks are really the physical elementary particles which baryons and mesons are made of [some recent experiments¹⁵⁾ in electro- and neutrino-production on nucleons give impressive evidence for this composite structure of "elementary" particles]. In this case, it is necessary (and, mathematically it is always possible) to take into account quark statistics when we consider composite systems like baryons.

It should seem natural to impose Fermi-Dirac statistics to quarks, as they behave like spin- $\frac{1}{2}$ particles. But there is good experimental evidence against this choice: fermion statistics give predictions for the ratio $(e^+e^- \rightarrow \text{hadrons}/e^+e^- \rightarrow \mu^+\mu^-)$ and for the decay $\pi^0 \rightarrow 2\gamma$ which are wrong at least by a factor of 3¹⁶⁾. Furthermore, when the "orbital" angular momentum $\vec{L}$ is introduced in the classification group, in order to account for higher spin states, then the stable octet and the lowest-lying decuplet baryons are assigned to a 56 $L = 0$; the 56 states are symmetric in the SU(6) indices, and Fermi-Dirac statistics would imply an antisymmetric $L = 0$ spatial configuration, which looks like too complicated a state to be lying lowest in mass.

Different schemes¹⁶⁾ have been proposed for quark statistics in order to allow for symmetric three-quark states (by the way, the above-mentioned experiments cannot discriminate between these theories yet). We just remark that from the point of view of baryon states they are all equivalent to impose Bose statistics and build symmetric three-quark systems.

From the permutational symmetry properties of the SU(6) baryon states, as given in Appendix II, we can deduce the corresponding symmetry of the space wave-functions:

SU(6) representation	Symmetry in SU(6) indices	Required spatial configuration
<u>56</u>	symmetric	symmetric
<u>70</u>	mixed	mixed
<u>20</u>	antisymmetric	antisymmetric

I.5 MIXING IN THE CLASSIFICATION

One of the more troublesome obstacles in our approach comes from the fact that the classification scheme considered here is not based on a good hadronic symmetry so that many of the defined quantum numbers are not expected to be conserved. Therefore states of the classification group with the same good quantum numbers can mix together in the physical states. At the present stage of our understanding, no theoretical skill exists to deal with this kind of mixing [no faithful approximation for the breaking, as the "octet enhancement" in $SU(3)$], and we have to content ourselves with a descriptive approach¹⁾ based on experimental data. In the following we give some illustrative examples of the kinds of mixing which can occur in the $SU(6) \times O(3)$ classification of hadrons:

- i) For mesons, as $SU(3)$ is a broken symmetry, the isoscalar member of an octet and a singlet can mix together; for example, in the $\underline{35} + \underline{1}$, $L = 0$, η and η' on one side, ω and ϕ on the other. It seems that the mesonic $SU(3)$ mixing angles are either very small ($\lesssim 10^\circ$, e.g. η - η') or near the "magic" mixing angle, ϕ_M ($\cos \phi_M = \sqrt{2}/\sqrt{3}$, e.g., ω - ϕ); interestingly enough, ϕ_M corresponds to the situation where one particle has only non-strange quarks and antiquarks (p, n and $\bar{p}, \bar{n}$) and the other only the strange ones ($\lambda, \bar{\lambda}$).
- ii) In the $\underline{35} + \underline{1}$ $L = 1$, G -parity prevents the A_1 and B to be mixed, but not their strange partners (Q_A and Q_B).
- iii) In a $\underline{70}$, $L = 1$ representation for baryons, we can form two $\underline{8}$'s with $J = \frac{3}{2}$, one with $S = \frac{1}{2}$, the other with $S = \frac{3}{2}$; the members of the physical octets can be mixtures of them.
- iv) In a $\underline{70}$, the $I = 1$ (Σ) states in the $S = \frac{1}{2}$ octet and decuplet can be mixed in the physical Σ 's; analogously for the $I = 0$ (Λ -) states in the $S = \frac{1}{2}$ octet and singlet.
- v) Even worse than that, the physical states can have components in different representations of $SU(6) \times O(3)$.

We remark that the mixing in the classification should not be confused with the chiral content of the states, which we discuss in

Chapters III, IV and V: the latter obeys different rules, it could have a different origin and it should be present even in the absence of the former. In this work, we give only a model for the chiral content of hadrons and the mixing in the classification will be introduced, if needed, on empirical grounds.

I.6 SU(6) AS A SYMMETRY

The underlying ideas, the successful predictions, and the failures of SU(6) as a hadronic symmetry are extensively discussed in Ref. 17. It was already born as a broken symmetry as even its subgroup SU(3) is so. It is also "static", i.e. meaningful only for particles at rest, as the "total quark-spin" ($\vec{S}$), defined in Section I.1, is conserved. Therefore, the most natural application would be to the hadron masses, and SU(6) turns out to be badly broken there, as the mass operator is far from being a scalar. In order to consider magnetic moments we have to introduce an operator and postulate its SU(6)-properties: if it is classified in a 35 representation, we get the famous $3/2$ value for the ratio between the proton and the neutron magnetic moments.

The generalizations of SU(6) do not seem very successful¹⁷⁾. Only the simplest one $SU(6)_q \times SU(6)_{\bar{q}}$, remains popular, together with its subgroups. Here, the first SU(6) refers only to "quarks", the second only to "antiquarks". Mesons are then classified in $(6, \bar{6})$ representations, baryons in $(\underline{56}, \underline{1})$ ($\underline{70}, \underline{1}$) or $(\underline{20}, \underline{1})$ ones. It is clear that as a classification group it is as good as SU(6), the only advantage arising from the fact that a 35 and a 1 come together in the $(\underline{6}, \bar{\underline{6}})$ of $SU(6)_q \times SU(6)_{\bar{q}}$. For baryons, the possibility of putting a 56-plet and a 70-plet of SU(6) simultaneously in a $(\underline{21}, \underline{6})$ representation of $SU(6)_q \times SU(6)_{\bar{q}}$ seems very unphysical.

I.7 THE $SU(6)_W$ GROUP¹⁷⁾

One interesting subgroup of $SU(6)_q \times SU(6)_{\bar{q}}$ is the so-called $SU(6)_W$. Its generators are $Q_{W\mu}^\alpha = (Q_\mu^\alpha)_q \times I_{\bar{q}} \pm I_q \times (Q_\mu^\alpha)_{\bar{q}}$, with (+) for $\mu = 0, 3$ and (-) for $\mu = 1, 2$. For example, the spin-operators, $\vec{S}$, of the static group

will correspond to the W-spin operators, $\vec{W}$ (in particular $W_z = S_z$). In the usual classification of baryons (which contains no antiquark) it clearly coincides with SU(6), but for mesons it is quite different: if we look again into the meson states for (static) SU(6), Section I.2, we shall find in the new $SU(6)_W$ that the $S = 1$, $S_z = \pm 1$ and the $S = 0$, $(\underline{8} + \underline{1})$ states of SU(6) come together in a $W = 1$ triplet, $(\underline{8} + \underline{1})$, while the $S = 1$, $S_z = 0$, $(\underline{8} + \underline{1})$ states will form a $W = 0$ $(\underline{8} + \underline{1})$ multiplet. We see that from the point of view of classification the two SU(6)'s, static and W, are equivalent being merely rearrangements inside the multiplets (with the minor difference in the assignment of a singlet). The important discrimination comes from the symmetry predictions, as a consequence of their different properties under Lorentz transformations. The static SU(6) is non-relativistic and can only be a symmetry of the S-matrix between states at rest. Instead, the $SU(6)_W$ is invariant under boosts in the z-direction and can be a symmetry of the S-matrix between collinear states: 2-body decays, forward scattering, total cross-sections. Indeed it has been widely applied to these reactions with moderate success (see Ref. 17). Its most famous prediction is the Johnson-Treiman relation:

$$\sigma_T(\pi^-p) - \sigma_T(\pi^+p) = \sigma_T(\bar{K}^0p) - \sigma_T(K^0p) = \frac{1}{2} \{ \sigma_T(K^-p) - \sigma_T(K^+p) \}$$

which is well verified experimentally¹⁷⁾.

Notice that in Section V.6 we get the same relation in a completely different approach. Indeed the Johnson-Treiman relation was obtained by imposing $SU(6)_W$ invariance and noticing that in the t-channel the difference $\sigma_T(Mp) - \sigma_T(\bar{M}p)$ contains only one amplitude, as well as in the s-channel, because we can use $SU(6)_W$ invariant crossing matrices. Instead, we obtain this relation from the contribution of the s-channel resonances, by assuming only SU(3) symmetry and, as a consequence, SU(3) crossing matrices.

Remark: The classification group discussed here will be called $SU(6)^{\text{class}}$ [or $SU(6)_W^{\text{class}}$] in the next chapters, in order to distinguish it from the $SU(6)_W^{\text{curr}}$ algebra for charges discussed in Chapter II. Also, the quarks discussed in section I.1 will be called constituent quarks.

CHAPTER II

SU(6)_W: THE ALGEBRA OF THE "GOOD" CHARGES

The algebra of currents ²⁾ has received much attention during the past eight years and it has given some fundamental results in hadronic physics. Here, we shall concentrate on the topics of this vast subject which are going to be used in the next chapters, and, mainly, on the SU(6)_W algebra for charges.

II.1 THE CHIRAL ALGEBRA OF WEAK AND ELECTROMAGNETIC CHARGES

By applying, in a first order approximation, the current-current interaction for weak processes, we can measure the matrix elements of the weak currents between hadrons. The V-A theory postulates vector and axial-vector pieces in the hadronic part of weak currents, in perfect analogy with the leptonic part. These are the strangeness conserving vector, $V_\mu^1(x) \pm i V_\mu^2(x)$, and axial, $A_\mu^1(x) \pm i A_\mu^2(x)$, currents, and the strangeness changing vector $V_\mu^4(x) \pm i V_\mu^5(x)$ and axial, $A_\mu^4(x) \pm i A_\mu^5(x)$, currents. We can define the corresponding weak vector and axial charges as ($\alpha = 1, 2, 4, 5$):

$$Q^\alpha(t) = \int d^3\vec{x} V_0^\alpha(x) \quad Q_S^\alpha(t) = \int d^3x A_0^\alpha(x) . \quad (II.1)$$

If the electromagnetic charge (in its Gell-Mann/Nishijima form)

$$Q^{EM} = I^3 + \frac{1}{2} Y = Q^3 + \frac{1}{\sqrt{3}} Q^8 \quad (II.2)$$

is added to this set, the principle of conserved vector currents (CVC) postulates that the strangeness conserving vector charges, Q^1 and Q^2 , together with the isovector part of Q^{EM} , Q^3 , are the generators of the isospin transformations

$$Q^i(t) = I^i \quad [I^i, I^j] = i\epsilon^{ijk} I^k \quad (II.3)$$

while $(2/\sqrt{3})Q^8 = Y$ is the hypercharge; these operators are then conserved. In order to incorporate the principle of universality of weak interactions between leptons and hadrons, this $SU(2) \times U(1)$ algebra is extended to $Q^4(t)$ and $Q^5(t)$ by assuming that all the measurable vector charges together close the $SU(3)$ algebra

$$[Q^\alpha(t), Q^\beta(t)] = if^{\alpha\beta\gamma} Q^\gamma(t) \quad (\alpha, \beta, \gamma = 1, 2, \dots, 8)$$

and these eight charges are identified to the generators of the $SU(3)$ (broken) symmetry (this identification can be considered merely as a good approximation based on the Ademollo-Gatto theorem; for a discussion on this point see, for instance, Ref. 6).

Finally, Gell-Mann has proposed that an $SU(3) \times SU(3)$ algebra is closed when we add also the axial charges, $Q_5^\alpha(t)$:

$$[Q^\alpha(t), Q^\beta(t)] = if^{\alpha\beta\gamma} Q^\gamma(t) \quad (II.4a)$$

$$[Q^\alpha(t), Q_5^\beta(t)] = if^{\alpha\beta\gamma} Q_5^\gamma(t) \quad (II.4b)$$

$$[Q_5^\alpha(t), Q_5^\beta(t)] = if^{\alpha\beta\gamma} Q^\gamma(t) . \quad (II.4c)$$

The charges $Q^1, Q^2, Q^3, Q^4, Q^5, Q^6, Q_5^1, Q_5^2, Q_5^4$ and Q_5^5 are directly measured in weak and electromagnetic interactions, while $Q^6, Q^7, Q_5^3, Q_5^6, Q_5^7$ and Q_5^8 are defined by the commutators in Eq. (4). The $SU(3) \times SU(3)$ structure generated by the vector and axial charges is called chiral algebra. Notice that the strangeness changing vector charges Q^4, Q^5, Q^6, Q^7 are only conserved in the limit of exact $SU(3)$ symmetry; the axial charges are not expected to be conserved. Therefore, we assume that the operators in the equal-time commutation relations, Eq. (II.4), have such a time-evolution as to preserve the chiral algebra.

This simple, beautiful pattern for the hadronic weak and e.m. charges has been tested in several ways ²⁾ and is verified to the accuracy of the present experiments. Therefore, Eq. (II.4) can be considered

at least as an excellent approximation; this validity of the chiral algebra is our basic hypothesis in this work.

II.2 THE SATURATION OF THE CHIRAL ALGEBRA IN THE $p_z \rightarrow \infty$ FRAME

The realization of the $SU(3) \times SU(3)$ algebra, Eq. (II.4), by hadronic states is called its saturation, and gives much valuable information on baryons and mesons. In order to see the kind of problem that we have to face in the saturation of the chiral algebra, we take the commutator in Eq. (II.4) between two identical one-particle states $|a, \vec{p}\rangle$:

$$\begin{aligned} \langle a, \vec{p} | [Q_5^\alpha(t), Q_5^\beta(t)] | a, \vec{p} \rangle &= i f^{\alpha\beta\gamma} \langle a, \vec{p} | Q_5^\gamma | a, \vec{p} \rangle = \\ &= \sum_n \left(\langle a, \vec{p} | Q_5^\alpha(t) | n, \vec{p} \rangle \langle n, \vec{p} | Q_5^\beta(t) | a, \vec{p} \rangle - \alpha \leftrightarrow \beta \right), \end{aligned} \quad (\text{II.5})$$

where we have introduced a complete set of hadronic states $\{n\}$. Now we consider the "mass" of the charges, $q^2 = (p_a - p_n)^2 = [(\mathbf{m}_a^2 + \vec{p}^2)^{\frac{1}{2}} - (\mathbf{m}_n^2 + \vec{p}^2)^{\frac{1}{2}}]^2$ and notice that 1) each matrix element is to be taken at a different positive value of q^2 , whereas the weak charges are measurable only for $q^2 \leq 0$; 2) these q^2 -values depend on $\vec{p}$, i.e., on the reference frame. The simplest way to overcome this difficulty is to go to a $p \rightarrow \infty$ reference frame, where $q^2 \rightarrow 0$ for finite values of $\mathbf{m}_n^2$. The advantages of the choice of this infinite momentum frame are then the following ²⁾:

- i) All the matrix elements $\langle a, \vec{\infty} | Q_5^\alpha(t) | n, \vec{\infty} \rangle$ are to be taken at $q^2 = 0$ for $\mathbf{m}_n^2 < \infty$.
- ii) Many-particle states (more precisely, states with $\mathbf{m}_n^2 \rightarrow \infty$ do not contribute to the sum rule, Eq. (II.5), in this limit.
- iii) As $q^2 = 0$ we can apply the PCAC hypothesis (see section II.4) to relate the axial charge matrix elements to pionic couplings.

The proofs of point (ii) are not completely rigorous ²⁾ and, in some sense, it must be considered as a reasonable assumption. Point (iii) is

very useful because the available neutrino beams are not good enough to provide us with precise results for the Q_5^α matrix elements.

The main subject of this work is the saturation of chiral algebra. On the basis of the discussion above we shall make two hypotheses in the saturation procedure:

- a) Eqs. (II.4) are realized by the one-particle hadronic states in the $p_z \rightarrow \infty$ frame (stated more precisely: the commutation rules are realized by the subspace of states which have a well-defined mass, i.e., stable particles and resonances).
- b) The resonance contributions are taken in the narrow width approximation (this is important in order to exploit point (iii) above).

Hereafter we shall always choose a reference frame such that $p_z \rightarrow \infty$. In this limit, the 4th components of the vector and the axial currents, V_0^α and A_0^α , are identical to the 3rd components, V_z^α and A_z^α , so that in a $p_z \rightarrow \infty$ frame,

$$Q^\alpha(t) = \int d^3\vec{x} V_z^\alpha(x) \quad Q_5^\alpha(t) = \int d^3\vec{x} A_z^\alpha(x). \quad (\text{II.6})$$

Eq. (6) will be used in the text as an equivalent definition of the charges, as long as we are stuck on a $p_z \rightarrow \infty$ frame.

II.3 GOOD AND BAD CHARGES; THE ALGEBRA OF GOOD CHARGES

We can consider other charges, defined as the space integral of the spatial components of the vector and the axial currents; an $SU(6) \times SU(6)$ algebra can be consistently postulated for the 72 charges that result from this extension¹⁸⁾. We can also define scalar, pseudoscalar and tensor currents, with corresponding charges, from their internal quantum numbers and their properties under Lorentz transformations¹⁸⁾. Together with the (72) vector and axial ones, these (72) charges will generate an $SU(6,6)$ algebra.

However, half of the charges under consideration have vanishing matrix elements between $p_z \rightarrow \infty$ one-particle states and are called bad; those with finite values in the $p_z \rightarrow \infty$ frame are the good ones. Therefore, the commutation rules involving bad charges cannot be saturated in a $p_z \rightarrow \infty$ frame¹⁸⁾ because either a) they give zero on both sides of the corresponding sum rules and no meaningful information at all, or, even worse, b) the sum rules are not defined in this limit.

In order to perform the saturation along the lines of the preceding section, we must consider only the subalgebra of the good charges (which clearly close one). In the $p_z \rightarrow \infty$ frame the good charges become identical by pairs; for example, Eqs. (II.1) and (II.6) are equivalent in this limit. Consequently, the interesting subalgebra is reduced, in the $p_z \rightarrow \infty$ frame, to the so-called $SU(6)_W^{\text{curr}}$ algebra of the good charges: vector (Q^α), axial (Q_5^α) and the following tensorial ones:

$$Q_\pm^\alpha = \int d^3\vec{x} \frac{1}{\sqrt{2}} \left\{ T_{01}^\alpha(x) \pm i T_{02}^\alpha(x) \right\}, \quad (\text{II.7})$$

where $T_{\mu\nu}(x)$ are tensor currents which have not yet been clearly associated with observable quantities and no theoretical prediction for the tensor charges could be compared to experiment without strong, dubious hypotheses. For this reason, we attach less importance to the physical consequences of the $SU(6)_W^{\text{curr}}$ algebra than the chiral $SU(3) \times SU(3)$ one. They are discussed here mostly for the sake of completeness.

II.4 PCAC

One of the most useful hypotheses in hadron physics is the one of a pion dominated divergence of the axial current²⁾, usually referred to as PCAC (partially conserved axial currents). It relates the strangeness conserving axial charge matrix elements at $q^2 = 0$ to the pionic couplings as follows (α and β are hadrons with momenta p and p' and helicity h)²⁾

$$\begin{aligned} \langle \beta, p', h; \pi^i, q = p - p' | T | \alpha, p, h \rangle_{q^2 = \mu^2} &= \\ &= \frac{(m_\alpha^2 - m_\beta^2)}{(2\pi)^{3/2} \sqrt{8p_0 p'_0} f_\pi} \langle \beta, p', h | Q_5^i | \alpha, p, h \rangle_{q^2 = 0}, \end{aligned} \quad (\text{II.8})$$

where f_π is the pion decay constant ($f_\pi \approx 95$ MeV). From Eq. (II.8) we deduce for the width of the decay $\alpha \rightarrow \beta + \pi^i$:

$$\Gamma(\alpha \rightarrow \beta \pi^i) = \frac{(m_\alpha^2 - m_\beta^2)^3 \sum_h |\langle \beta, h | Q_5^i | \alpha, h \rangle|^2}{16 \pi f_\pi^2 m_\alpha^3 (2J_\alpha + 1)}. \quad (\text{II.9})$$

(From now on, the Q_5^α matrix elements are always taken at $q^2 = 0$).

Eq. (II.9) will be used in order to extract the values of the axial charge matrix elements from the experimental decay widths of mesons and baryons.

By exploiting in a suitable way Eq. (II.8), Adler and Weisberger transformed the commutator of two axial charges, Eq. (II.4c), sandwiched between proton states in a very convergent sum rule for total cross-sections ²⁾:

$$1 = G_A^2 + \frac{f_\pi^2}{\pi} \int_{m_\pi}^{\infty} dE_\pi^L \frac{p_\pi^L}{(E_\pi^L)^2} [\sigma_T(\pi^- p) - \sigma_T(\pi^+ p)]_{E_\pi^L} \quad (\text{II.10})$$

($G_A \approx 1.23$ is the β -decay constant).

In our approach, this relation should be saturated by the resonance contributions to the cross-sections. By taking the experimental values of the pionic couplings we can verify that this is a very good approximation. Also, on more theoretical grounds, the duality hypothesis ¹⁹⁾ predicts that the combination of cross-sections in Eq. (II.10) correspond to the contribution of resonances only, as it is odd under charge conjugation.

CHAPTER III

THE CHIRAL CONTENT OF HADRONS

III.1 TWO SU(6)'s

We have seen that $SU(6)^{\text{class}}$ [or $SU(6)_W^{\text{class}}$] seems a good classification group for hadrons and, perhaps, a broken symmetry for hadronic reactions. On the other hand, the good charges define an $SU(6)_W^{\text{curr}}$ or its chiral $SU(3) \times SU(3)$ subgroup if we want to take only vector and axial charges into account. The vector charges define an $SU(3)$; CVC with the Cabibbo theory works so well, that we can conclude that it is a good approximation to put $SU(3)^{\text{curr}} = SU(3)^{\text{class}}$, i.e., their corresponding generators are the same operators. Can we generalize this to $SU(6)_W^{\text{curr}} = SU(6)_W^{\text{class}}$? In other words, can we identify the generators of both $SU(6)$'s with each other? No, it has been known for a long time that they cannot be the same. To make this identification we should avoid quite general no-go theorems like Coleman's²⁰⁾. But we have also conclusive experimental evidence that hadrons must have components in many different multiplets of $SU(6)_W^{\text{curr}}$. For instance, if we consider the commutator of two axial charges, Eq. (II.4c), between nucleon states [which, assuming PCAC, is connected by the Adler-Weisberger relation, Eq. (II.10), to experimental quantities] we know that it receives contributions from many different resonances, so that the nucleons cannot belong to a simple representation of the chiral $SU(3) \times SU(3) \subset SU(6)_W^{\text{curr}}$, as it belongs to a $(\underline{6}, \underline{3})$ of the corresponding $SU(3) \times SU(3) \subset SU(6)_W^{\text{class}}$. Also, the π and the ρ are classified in a $\underline{35}$ of $SU(6)_W^{\text{class}}$, and if it coincides with $SU(6)_W^{\text{curr}}$, the ρ -width should be about $2\frac{1}{2}$ times the experimental value, as they alone have to saturate some of the charge commutators (PCAC being used in order to relate the axial charge matrix elements to the pionic couplings). The disagreements are too large to be explained by a failure of PCAC, but even by looking directly at G_A , the axial charge matrix between nucleons, "SU(6) symmetry" [in the sense of identifying $SU(6)_W^{\text{curr}} = SU(6)_W^{\text{class}}$] predicts $G_A = \frac{5}{3}$, while experimentally, $G_A \approx 1.2$. On more

theoretical grounds, the anomalous magnetic moment of the nucleons should vanish (see Appendix III).

We could go on and find lots of examples from which we can clearly conclude that hadrons must have components in many different representations of the $SU(6)_W^{\text{curr}}$. Stated otherwise, each hadron belongs to a highly reducible representation of $SU(6)_W^{\text{curr}}$: the reduction of this representation into irreducible ones will give the chiral contents of the hadron (we adopt the term chiral because the more relevant part of $SU(6)_W^{\text{curr}}$, as already mentioned in Chapter II, is the chiral subgroup). Hadrons, as classified in $SU(6)^{\text{class}}$ are simple systems of constituent quarks (see Chapter I), and, analogously, the multiplets of $SU(6)_W^{\text{curr}}$ are simply built from the current quarks, which correspond to its fundamental representation. Therefore, we can suggestively speak of the chiral content as the current quark content of hadrons; it should be rather complex, in opposition to the simple constituent quark content.

III.2 CONFIGURATION MIXING

Once we realize that we cannot identify the two $SU(6)$'s, we get a new problem: to discover the chiral content of hadrons. This means that we have to express hadrons in the base where $SU(6)_W^{\text{curr}}$ is diagonal, and in this new base, each $SU(6)_W^{\text{class}}$ state will appear as a superposition of many different basic states. In this sense, we say we have a "mixing" of states, and what we are looking for is the mixing operator^{*)}: the transformation which passes from one base to the other.

There have been many attempts to solve this problem^{21, 22)}. In the beginning, people tried a simple approach by considering mixing inside sets of a few hadrons. They obtained some good results in this way, but as soon as inconsistencies appeared they had to introduce more and more states into their scheme and, consequently, more free parameters²²⁾. The

*) A "mixing operator" is a unitary transformation. There is nothing but simplicity to tell us that the transformation we are looking for is unitary: the only thing we need is to keep the $SU(6)$ algebra and this is guaranteed by a similitude transformation.

new approach to this problem, the one which we will be concerned with, is much more ambitious: we construct a simple mixing operator with as few parameters as possible, trying to guess and to understand the symmetry properties of this operator.

III.3 THE GELL-MANN/MELOSH PROGRAM

There is still more trouble with $SU(6)$ schemes which we have to worry about: in considering a broken symmetry it is important to know a (hypothetical or real) limit, or a simple model, where the exact symmetry is realized. For example, in the case of $SU(3)$, the free quark Hamiltonian is an $SU(3)$ -scalar in the limit of equal masses for the three quarks p , n , λ . This is not the case for $SU(6)_W^{\text{curr}}$; the kinetic energy of the free-quark is not invariant under this group as we shall see in the next section. Therefore, the meaning of a possible $SU(6)_W$ -symmetry remained obscure for years (and, consequently, the origin of a broken symmetry mixing).

Recently, an interesting work by Melosh⁸⁾ has put some lights on all that: it gives hints for both the chiral content of hadrons and the possibility of realizing an exact $SU(6)_W^{\text{class}}$ symmetry in the free-quark model. Melosh has shown that there exists a set of operators which:

- a) close an $SU(6)_W$ algebra;
- b) commute with the free-quark Hamiltonian;
- c) are invariant under Lorentz boosts in the z -direction; and
- d) are related to $SU(6)_W^{\text{curr}}$ by a unitary transformation.

The existence of these operators implies:

- i) they define an $SU(6)_W^{\text{class}}$ group which is an exact symmetry of the free-quark model;
- ii) the unitary transformation (mixing operator) relating them to the $SU(6)_W^{\text{curr}}$ charges gives the current quark content of the $SU(6)_W^{\text{class}}$ states (which here are the constituent quarks).

III.4 THE MELOSH TRANSFORMATION

We shall not enter into the reasoning followed by Melosh to construct such a unitary operator, but rather summarize his results. One introduces the free-quark field $q(x)$ describing a Dirac particle in an $SU(3)$ triplet. It obeys the equal-time anti-commutation rules:

$$\{q_\alpha(x), q_\beta^\dagger(y)\}_{x_0=y_0} = \delta_{\alpha\beta} \delta^3(\vec{x}-\vec{y}) \quad (\text{III.1})$$

$$\{q_\alpha(x), q_\beta(y)\} = \{q_\alpha^\dagger(x), q_\beta^\dagger(y)\} = 0.$$

The indices α, β run through the six possible states of spin and unitary-spin of quarks (see section I.1). The Hamiltonian, in the case of an $SU(3)$ -invariant quark mass, is ($\vec{p}_\perp = \vec{p} \times \hat{z}$)

$$H = \int d^3\vec{x} q^\dagger(x) (\vec{\alpha} \cdot \vec{p}_\perp + \alpha_z p_z + \beta m) q(x). \quad (\text{III.2})$$

For further use, consider also the spin, the orbital angular momentum and the Lorentz-boosting operators:

$$\begin{aligned} \vec{J} &= \int d^3\vec{x} q^\dagger(x) \frac{\vec{\sigma}}{2} q(x) & \vec{L} &= \int d^3\vec{x} q^\dagger(x) (\vec{x} \times \vec{p}) q(x) \\ \vec{K} &= \int d^3\vec{x} q^\dagger(x) \vec{\alpha} q(x). \end{aligned} \quad (\text{III.3})$$

The $SU(6)_W$ charges are obtained by the usual procedure: we define suitable infinitesimal transformations of the quark field and the corresponding generators are obtained through Noether's theorem. From the Hamiltonian of Eq. (III.2) one gets:

$$\begin{aligned} Q^\alpha &= \int d^3\vec{x} q^\dagger(x) \frac{\lambda^\alpha}{2} q(x) \\ Q_5^\alpha &= \int d^3\vec{x} q^\dagger(x) \frac{\sigma_z \lambda^\alpha}{2} q(x) \\ Q_\pm^\alpha &= \int d^3\vec{x} q^\dagger(x) \frac{\beta \sigma_\pm \lambda^\alpha}{2} q(x). \end{aligned} \quad (\text{III.4})$$

Now, from Eqs. (III.1) to (III.5), one verifies that:

- i) only Q^α commutes with H (and also with $\vec{S}, \vec{L}, \vec{K}$); this gives the $SU(3)$ symmetry of the free-quark model with an $SU(3)$ -scalar mass operator;
- ii) the other Q 's commute with the $\alpha_z p_z$ and βm terms in H , but anti-commute with the $\vec{\alpha} \cdot \vec{p}_\perp$ term. Notice that all the Q 's commute with K_z and $\vec{L}$, but not with $K_\pm$.

Then, Melosh defines the following unitary transformation (which is the analogous of a Foldy-Wouthusen transformation for the transverse directions only) ⁸⁾:

$$V = e^{iY} \quad Y = \frac{1}{2} \int d^3\vec{x} q^\dagger(x) \left(\arctan \frac{\vec{\sigma} \cdot \vec{p}_\perp}{m} \right) q(x) \quad (\text{III.5})$$

and he considers the new operators:

$$\tilde{Q}^\alpha = V Q^\alpha V^\dagger = Q^\alpha$$

$$\tilde{Q}_5^\alpha = V Q_5^\alpha V^\dagger = \int d^3\vec{x} q^\dagger(x) \left\{ \frac{m}{\omega} \sigma_z - \frac{(\vec{\gamma} \times \vec{p})_z}{\omega} \right\} \frac{\lambda^\alpha}{2} q(x) \quad (\text{III.6})$$

$$Q_\pm^\alpha = V Q_\pm^\alpha V^\dagger = \int d^3\vec{x} q^\dagger(x) \left\{ \beta \sigma_\pm + \left[\gamma_5 - \frac{\beta(\vec{\sigma} \cdot \vec{p}_\perp)}{\omega + m} \right] \frac{p_\pm}{\omega} \right\} \frac{\lambda^\alpha}{2} q(x),$$

where $\omega = \sqrt{p_\perp^2 + m^2}$.

Now, by using Eqs. (III.1), (III.2), (III.3) and (III.6), it is straightforward to verify that the charges $\tilde{Q}$ verify the properties (a) to (d) above:

- a) the $SU(6)_W$ algebra of the Q 's is conserved as $V^\dagger = V^{-1}$;
- b) $[H, \tilde{Q}] = 0$, for all $\tilde{Q}$'s in Eq. (III.6);
- c) $[K_z, V] = 0$, $[K_z, Q] = 0 \rightarrow [K_z, \tilde{Q}] = 0$ for all Q 's;
- d) by definition $\tilde{Q} = V Q V^\dagger = V Q V^{-1}$.

We shall further discuss the consequences of the Melosh transformation in the next section, but before let us do some mathematical manipulations for later use. First, notice that we can introduce

$$\tilde{q}(x) = V q(x) V^\dagger \quad (\text{III.7})$$

and in this new base, the $\tilde{Q}$'s are analogous to the Q 's in the $q(x)$ base:

$$\begin{aligned} \tilde{Q}^\alpha &= \int d^3\vec{x} \tilde{q}^\dagger(x) \frac{\lambda^\alpha}{2} \tilde{q}(x) \\ \tilde{Q}_5^\alpha &= \int d^3\vec{x} \tilde{q}^\dagger(x) \frac{\sigma_z \lambda^\alpha}{2} \tilde{q}(x) \\ \tilde{Q}_\pm^\alpha &= \int d^3\vec{x} \tilde{q}^\dagger(x) \frac{\beta \sigma_\pm \lambda^\alpha}{2} \tilde{q}(x) \end{aligned} \quad (\text{III.8})$$

but the Hamiltonian becomes,

$$H = \int d^3\vec{x} \tilde{q}^\dagger(x) \left\{ \alpha_z p_z + \beta \sqrt{p_\perp^2 + m^2} \right\} \tilde{q}(x) \quad (\text{III.9})$$

which is similar to the Foldy-Wouthusen Hamiltonian. Also the $\vec{S}$ and $\vec{L}$ operators will have different expressions in this basis. For example,

$$S_z = \frac{1}{2} \int d^3\vec{x} \tilde{q}^\dagger(x) \left\{ \frac{m}{\omega} \sigma_z - \frac{(\vec{r} \times \vec{p})_z}{m} \right\} \tilde{q}(x). \quad (\text{III.10})$$

In the next sections, we shall prefer to work in the momentum representation. We introduce creation and annihilation operators in the momentum space, with a box normalization:

$$q(x) = \frac{1}{(2\pi)^{3/2}} \sum_{\vec{p}\alpha s} \left\{ e^{-ipx} a_{\vec{p}}^{(s,\alpha)} u_{\vec{p}}^{(s,\alpha)} + e^{ipx} b_{\vec{p}}^{\dagger(s,\alpha)} v_{\vec{p}}^{(s,\alpha)} \right\} \quad (\text{III.11})$$

where α is an SU(3) index and the notation is standard elsewhere. In this representation Eq. (III.6) reads

$$\tilde{Q}^\alpha = \sum_{\vec{p}} \left\{ q^\dagger(\vec{p}) \frac{\lambda^\alpha}{2} q(\vec{p}) - \bar{q}^\dagger(\vec{p}) \frac{\lambda^\alpha}{2} \bar{q}(\vec{p}) \right\}$$

$$\tilde{Q}_5^\alpha = \sum_{\vec{p}} \left\{ q^\dagger(\vec{p}) V(\vec{p}) \frac{\lambda^\alpha \sigma_z}{2} V^\dagger(\vec{p}) - \bar{q}^\dagger(\vec{p}) V(\vec{p}) \frac{\lambda^\alpha \sigma_z}{2} V^\dagger(\vec{p}) \bar{q}(\vec{p}) \right\}$$

(III.12)

$$\tilde{Q}_\pm^\alpha = \sum_{\vec{p}} \left\{ q^\dagger(\vec{p}) V(\vec{p}) \frac{\lambda^\alpha \sigma_\pm}{2} V^\dagger(\vec{p}) q(\vec{p}) + \bar{q}^\dagger(\vec{p}) V(\vec{p}) \frac{\lambda^\alpha \sigma_\pm}{2} V^\dagger(\vec{p}) \bar{q}(\vec{p}) \right\},$$

where $q(\vec{p}) = \sum_{s,\alpha} a_{\vec{p}}^{(s,\alpha)} f^{(s,\alpha)}$, etc., and $f^{(s,\alpha)}$ is a Pauli spinor, SU(3) triplet, i.e., a vector in a six-dimensional SU(6) space (a quark spinor). The unitary transformation

$$V(\vec{p}) = e^{i\theta' \frac{(\vec{\sigma} \times \vec{p}_\perp)_z}{p_\perp}} \quad \theta' = \arctan \frac{p_z p_\perp}{(E+\omega)(\omega+m)}$$

(III.13)

$$\omega = \sqrt{p_\perp^2 + m^2} \quad E = \sqrt{p^2 + m^2}.$$

Finally, notice that in a $p_z \rightarrow \infty$ frame,

$$U(\vec{p}_\perp) = \lim_{p_z \rightarrow \infty} V(\vec{p}) = e^{i\theta \frac{(\vec{\sigma} \times \vec{p}_\perp)_z}{2p_\perp}} \quad \theta = \arctan \frac{p_\perp}{m}. \quad (III.14)$$

III.5 PHYSICAL CONSEQUENCES

Although the free-quark model has little to do with reality, the Melosh transformation has at least two qualities:

- i) a theoretical interest, as it solves the puzzle of SU(6) breaking and configuration mixing in a model;
- ii) a phenomenological application, as a guide for understanding the properties of charges between states classified in SU(6) multiplets.

In our work we shall be concerned with point (ii). The usual approach (as mentioned in the Introduction) is:

- a) to look at the transformation properties of the different pieces appearing in the operators in the free-quark model, and
- b) to postulate a similar structure for the physical operators.

The credibility of this approach has already been discussed in the Introduction. Let us see how it works here and consider the axial charges in the light of the Melosh transformation.

From Eqs. (III.1), (III.3), (III.4), (III.5) and (III.6) we can easily verify that:

- 1) The Y operator behaves as an 35 object under both $SU(6)_W^{\text{curr}}$ and $SU(6)_W^{\text{class}}$, a vector under spin rotations, with $S_z = \pm 1$ and a tensor with $L_z = \pm 1$ under spatial rotations;
- 2) the(axial) charge $\tilde{Q}_5^\alpha$ has two pieces; both are 35 under $SU(6)^{\text{class}}$, the first is $S_z = L_z = 0$, with $L = 0, 2, 4 \dots$ and the second is $S_z = -L_z = \pm 1$, $L = 1, 3, 5, \dots$.

Some people¹⁰⁾ have assumed that the hadron axial charge has the same structure of two 35 operators, one being $S_z = L_z = 0$ and the other, $S_z = -L_z = \pm 1$. For mesons, the definition of the charge conjugation as $C = (-)^{L+S}$ uniquely determines these two operators. (The generality of the scheme increases and, as a trivial consequence, the predictability decreases, if the quantum number L is not considered as well-defined.) The results following for the charge matrix elements¹⁰⁾ are in good agreement with experiments and are much the same that we shall obtain with our formalism (as long as we look only for the low-lying $L = 0, 1$ hadron states). There is a problem: the same matrix elements can be obtained with another operator, which is more complicated than a 35, the only difference being some relative signs that are not yet available from experiments.

As baryons are classified in 56, 70, and 20 representations, we have three different 35 operators which, as a charge conjugation says nothing here, can be combined arbitrarily in both of the two postulated pieces of the axial charge. The choice of these 35's gives constraints on the baryon spectrum⁹⁾ as discussed in Chapter V. However, some results are generally true (see Chapter V): G_A is scaled down from its $SU(6)$ value $5/3$,

while the $SU(6)$ prediction for G^*/G_A and $(\frac{1}{2}^+$ octet) D/F ratios remain, everything in agreement with experiment.

III.6 OUR APPROACH: HADRONS AS QUARK SYSTEMS

We have already discussed on general grounds the motivation and some advantages of our formalism in the Introduction. Let us formulate here the basic hypotheses of the simple model we are going to construct.

The charge algebra is supposed to be saturated by one-particle states moving with infinite momenta (as mentioned in Chapter II). Therefore, we have to construct meson and baryon states moving in the z -direction with infinite momenta. They should belong to 1, 35 and 56, 70, 20 representations of the assumed classification group $SU(6) \times O(3)$. The simplest picture one can imagine is the following ⁹⁾

- A) $(p_z \rightarrow \infty)$ hadrons are composite systems of a few constituent quarks and antiquarks, all moving with infinite momentum in the z -direction (as hadrons do), i.e., $p_{zi} \rightarrow \infty$, and with transverse momenta $\vec{p}_{\perp i} = \vec{k}_i$ which must sum up to zero, as the composite systems have only longitudinal momentum;
- B) as a first approximation, the charges couple independently to the constituent quarks; we assume that the content in current quarks of the constituents is given by the Melosh transformation.

Under these assumptions, and from the point of view of the charge matrix elements, a hadron state can now be written as the tensor product of the states describing the constituent quarks and antiquarks (marked $\sim$ as for the physical hadron states):

$$|\widetilde{H}, p_z \rightarrow \infty\rangle = |\widetilde{q}_1, \vec{k}_1, p_{z1} \rightarrow \infty\rangle \dots |\widetilde{q}_N, \vec{k}_N, p_{zN} \rightarrow \infty\rangle. \quad (\text{III.15})$$

The current quark content of the hadron is then obtained by Melosh transforming each constituent. Notice that, as $p_{zi} \rightarrow \infty$ we can use Eq. (III.14) and omit p_{zi} in the states hereafter:

$$|\tilde{H} p_z \rightarrow \infty\rangle = U_1(\vec{k}_1) |q_1 \vec{k}_1\rangle \dots U_N(\vec{k}_N) |q_N \vec{k}_N\rangle. \quad (\text{III.16})$$

From hypothesis (B), the charge operators can be written in the current quark base as:

$$Q(|q_1 \vec{k}_1\rangle \dots |q_N \vec{k}_N\rangle) = \left(\sum_{i=1}^N Q_i\right) |q_1 \vec{k}_1\rangle \dots |q_N \vec{k}_N\rangle, \quad (\text{III.17})$$

where each Q_i applies only on the current quark (antiquark) q_i and is given by Eq. (III.4).

Notice that our hypotheses lead us to a scheme where the current quark content of each constituent is almost independent of the state of the others: the only constraint is trivial and kinematical, $\sum_i \vec{k}_i = 0$, but we shall see that it is enough to put some restrictions on charges: the conservation of charge conjugation for mesons and of the quark statistics for baryons⁹⁾.

To understand the rôle that interactions can play, we remember that in the free-quark model considered by Melosh, the $SU(6)_W^{\text{class}} \times O(3)$ is an exact symmetry; for example the $SU(6)_W^{\text{curr}}$ algebra is saturated at any $\vec{p}$ by one-quark states, all of them with mass m , so that $q^2 = 0$ (see Chapter II). But $SU(6)_W^{\text{class}} \times O(3)$ is badly broken for hadronic interactions and the states contributing to the sum rule, Eq. (II.5) have quite different masses. However, we might still think that only the transverse kinetic energy of constituents is responsible for the complex current quark structure of hadrons, which would then arise from the Melosh transformation of the constituents. Then the interactions should break the $SU(6)_W \times O(3)$ symmetry²³⁾. Nevertheless we can do better than that: interactions could manifest themselves by correlations between quarks, in such a way that each U_i in Eq. (III.16) could depend on the states of quarks other than q_i . In the formalism used here this could be simply performed and eventually explained from a physical, or at least an intuitive, point of view. Indeed, this should be the next step in the study of

the chiral content of hadrons within our approach. For the moment, only the "uncorrelated" case is considered and we shall see that the results are very encouraging.

CHAPTER IV

MESONS AS QUARK SYSTEMS

IV.1 THE CHIRAL CONTENT OF MESONS

In this chapter, the ideas which have previously been discussed are applied in the calculation of the axial charge matrix elements (pionic couplings) between meson states. In the spirit of hypotheses (A) and (B) of Chapter III we shall consider $p_z \rightarrow \infty$ meson states as made of a constituent quark-antiquark pair

$$|\tilde{M}\rangle = |\tilde{M}_{p_z \rightarrow \infty}\rangle = \sum_{\vec{k}} M(\vec{k}) |q \vec{k}\rangle |\bar{q} -\vec{k}\rangle, \quad (\text{IV.1})$$

where we have imposed zero transverse-momentum to the meson composite system; $M(\vec{k})$ is some distribution in the quark momenta which should be related to the meson quantum numbers (for example, L, L_z when defined) but for the time being we have not to worry about this. Notice that we mark $\sim$ the physical states (for example, $|\tilde{\pi}\rangle$ for the physical pion) supposed to be simple systems of constituent quarks ($\tilde{q}$). We shall consider also "mathematical" states

$$|M\rangle = \sum_{\vec{k}} M(\vec{k}) |q \vec{k}\rangle |\bar{q} -\vec{k}\rangle \quad (\text{IV.2})$$

which correspond to Eq. (IV.1) for current quark-antiquark pairs, so transforming as a 35 state under $SU(6)_W^{\text{curr}}$. Instead, $|\tilde{M}\rangle$ is to be classified in a 35 representation of $SU(6)_W^{\text{class}}$, as mentioned in Chapter I. The chiral content of a state $|\tilde{M}\rangle$ is obtained by applying Eq. (III.16)

$$\begin{aligned} |\tilde{M}\rangle &= \sum_{\vec{k}} (u(\vec{k}) |q \vec{k}\rangle) (\bar{u}(-\vec{k}) |\bar{q} -\vec{k}\rangle) \\ &\equiv e^{i\theta(k)Z} |M\rangle. \end{aligned} \quad (\text{IV.3})$$

Operators acting on antiquarks are marked — ; we use the same symbol for the operator $\vec{k}$ [as in $\theta(k)$] and its "eigenvalues" [as in $M(\vec{k})$]. From Eq. (III.14) one gets ⁹⁾

$$Z = \frac{[(\vec{\sigma} - \vec{\bar{\sigma}}) \times \vec{k}]_z}{2k} \quad \theta(k) = \arctan \frac{k}{m}. \quad (\text{IV.4})$$

We shall discuss the group theoretical properties of the θZ operator in the next section. First, let us look at the chiral content of mesons as given by Eq. (IV.3). Meson states are explicitly constructed in Appendix I (see also Chapter I). In a $\underline{35} + \underline{1}$ of $SU(6)$ we find $(\underline{8} + \underline{1})$ pseudoscalar mesons, P^α ($\alpha = 0, 1, 2, \dots, 8$) and 9 vector mesons V_h^α ($\alpha = 0, 1, 2, \dots, 8$; $h = +1, 0, -1$), where α is an $SU(3)$ index and h is the meson helicity. From Eqs. (IV.3) and (IV.4) we obtain the following chiral content for these $\underline{35}$ states with any spatial configuration

$$|\tilde{V}_{+1}^\alpha\rangle = \left(\frac{\omega+m}{2\omega}\right) |V_{+1}^\alpha\rangle - \frac{k_+ k_+}{\omega(\omega+m)} |V_{-1}^\alpha\rangle + \frac{k_+}{\omega} |P^\alpha\rangle \quad (\text{IV.5})$$

$$|\tilde{V}_0^\alpha\rangle = |V_0^\alpha\rangle \quad (\omega = \sqrt{k^2 + m^2})$$

$$|\tilde{P}^\alpha\rangle = \frac{m}{\omega} |P^\alpha\rangle - \frac{k_-}{\omega} |V_{+1}^\alpha\rangle - \frac{k_+}{\omega} |V_{-1}^\alpha\rangle.$$

From this we can calculate the chiral content of any hadron, once we know its constituent-quark configuration. As mentioned in Appendix I the P^α 's and V_h^α 's are classified under the chiral $SU(3) \times SU(3) \subset SU(6)_W^{\text{curr}}$ as follows

$$P^\alpha \in [(\underline{8}, \underline{1}) + (\underline{1}, \underline{8})] \quad P^0 \in (\underline{1}, \underline{1})$$

$$V_0^\alpha \in [(\underline{8}, \underline{1}) - (\underline{1}, \underline{8})] \quad V_0^0 \in (\underline{1}, \underline{1})^i$$

$$V_{\pm 1}^0, V_{\pm 1}^\alpha \in [(\underline{3}, \underline{\bar{3}}) - (\underline{\bar{3}}, \underline{3})].$$

Notice that the $\tilde{V}_0^\alpha$ do not mix in our simple scheme: this fact has several consequences for axial charge matrix elements which are approximately verified by experiment (see next sections). However, this non-mixing property of $\tilde{V}_0^\alpha$ puts some difficulties when enlarged algebraic schemes are considered, where other operators like the mass are included⁵⁾. Now, it is clear that, to cure this disease one could take some quark correlations into account, as already mentioned in Chapter III.

IV.2 ALGEBRAIC PROPERTIES OF THE AXIAL CHARGES

From Eq. (III.18), the axial charges act on current $q\bar{q}$ pairs as

$$Q_5^\alpha |q\vec{k}\rangle |\bar{q}-\vec{k}\rangle = X^\alpha |q\vec{k}\rangle |\bar{q}-\vec{k}\rangle \quad X^\alpha = \frac{1}{2} (\lambda^\alpha \sigma_z + \bar{\lambda}^\alpha \bar{\sigma}_z) \quad (\text{IV.6})$$

and, from Eqs. (IV.3) and (IV.4), the axial charge matrix elements between physical meson states are

$$\langle \tilde{M} | Q_5^\alpha | \tilde{M} \rangle = \langle M | e^{-i\theta Z} X^\alpha e^{i\theta Z} | M \rangle.$$

By defining an operator

$$Y^\alpha = \frac{\vec{k} \cdot (\lambda^\alpha \vec{\sigma} - \bar{\lambda}^\alpha \vec{\sigma})}{2k} \quad (\text{IV.7})$$

we notice the following commutation relations

$$[Z, X^\alpha] = -i Y^\alpha \quad [Z, Y^\alpha] = i X^\alpha \quad (\text{IV.8})$$

so that the charge takes the simple form⁹⁾

$$\langle \tilde{M} | Q_5^\alpha | \tilde{M} \rangle = \langle M | (X^\alpha \cos \theta - Y^\alpha \sin \theta) | M \rangle. \quad (\text{IV.9})$$

It is worth noticing that if we define

$$Z^\alpha = \frac{\lambda^\alpha [(\vec{\sigma} - \vec{\bar{\sigma}}) \times \vec{k}]_z}{2k} \quad (\text{IV.10})$$

the following SU(6) algebra is closed:

$$\begin{aligned} [Q^\alpha, Q^\beta] &= i f^{\alpha\beta\gamma} Q^\gamma \\ [Q^\alpha, X_i^\beta] &= i f^{\alpha\beta\gamma} X_i^\gamma \\ [X_i^\alpha, X_j^\beta] &= i \epsilon_{ijk} \left(\frac{2}{3} \delta_{\alpha\beta} X_k^0 + d^{\alpha\beta\gamma} X_k^\gamma \right) + i \delta_{ij} f^{\alpha\beta\gamma} Q^\gamma \end{aligned} \quad (\text{IV.11})$$

with $X_1^\alpha = X^\alpha$, $X_2^\alpha = -Y^\alpha$, $X_3^\alpha = Z^\alpha$ ($\alpha = 0, 1, \dots, 8$ and $\lambda^0 = I$); $Q^\alpha = \lambda^\alpha/2$ are the vector charges. Furthermore, if we consider only the isospin chiral algebra $SU(2) \times SU(2)$, an $O(5)$ subalgebra of this $SU(6)$ is closed:

$$\begin{aligned} [I^a, I^b] &= i \epsilon_{abc} I^c & [Z, Y^a] &= i X^a \\ [I^a, Y^b] &= i \epsilon_{abc} Y^c & [Z, X^a] &= -i Y^a \\ [I^a, X^b] &= i \epsilon_{abc} X^c & [X^a, Y^b] &= -i \delta_{ab} Z \\ [X^a, X^b] &= [Y^a, Y^b] = i \epsilon_{abc} I^c & [I^a, Z] &= 0 \quad (a, b = 1, 2, 3). \end{aligned} \quad (\text{IV.12})$$

Therefore the scheme shows some mathematical elegance which, eventually, could be further exploited.

Now let us concentrate on the $SU(6)_W^{\text{curr}}$ properties of the operators appearing in Eqs. (IV.6) to (IV.9). Remembering that X^α is nothing else but the axial charge in the current quark representation, we deduce that θZ and Y^α belong to a 35 multiplet of $SU(6)_W^{\text{curr}}$ and to a $(\bar{3}, 3) + (3, \bar{3})$ representation of its chiral subgroup; the latter decomposes into an $SU(3)$ -singlet (Z) and an $SU(3)$ -octet (Y^α), both with $S_z = \pm 1$. Furthermore,

under spatial $O(3)_L$ rotations $X \cos \theta$ has $L_z = 0$, $L = 0, 2, 4, \dots$; $Y \sin \theta$ and θZ have $L_z = \pm 1$, $L = 1, 3, 5, \dots$. These are exactly the same properties as for the corresponding operators in the free-quark model (see Chapter III), and we can now understand why our scheme will give much the same results as the "extrapolative" approach¹⁰.

Finally, we would like to stress an important property of the operator θZ : it commutes with the charge conjugation operator for a $\bar{q}q$ pair, $C = (-)^{L+S}$, as required from the well-defined $C = +1$ associated with the axial charge. We can trace this property back to the particular combination of two Melosh transformations in $e^{i\theta Z}$ which follows from the prescription of zero transverse momentum for the hadron state, or, equivalently, from taking only the relative quark motion into account.

IV.3 PIONIC COUPLINGS

By using the ($L = 0$) and ($L = 1$) meson states as given in Appendix I, the calculation of the axial charge matrix elements from Eq. (IV.9) is straightforward. First, we notice that the results will have $SU(3)$ symmetry as θZ is an $SU(3)$ scalar. Secondly, the matrix elements of axial charges between mesons are not directly measured, and we must use PCAC (see Chapter II) in order to compare them with experiment. The PCAC hypothesis is expected to work for pions, whose couplings are related therein to the strangeness conserving axial charges. Therefore, only the matrix elements of Q_5^j ($j = 1, 2, 3$) between non-strange mesons are relevant, as $SU(3)$ symmetry has not yet been broken in our axial charges. We introduce the following reduced matrix elements:

$$\langle N_h^i | Q_5^j | M_h \rangle = \delta_{ij} G_{MN}(h) \quad \text{if } \begin{cases} M \text{ has } I=0 \\ N \text{ has } I=1 \end{cases} \quad (\text{IV.13})$$

$$\langle N_h^i | Q_5^j | M_h^k \rangle = i \epsilon^{ijk} G_{MN}(h) \quad \text{if } \begin{cases} \text{both } M, N \\ \text{have } I=1 \end{cases}$$

$$(i, j = 1, 2, 3) \quad h = \text{helicity} = L_z + S_z.$$

The calculated G's are then given in Table IV.1. We assume a pure octet η . If we take into account $\approx 15\%$ of singlet contamination, the predicted values for $G_{A_2\eta}$ and $G_{\delta\eta}$ should be multiplied by a factor $\approx \frac{3}{2}$ if the mixing angle is positive and $\approx \frac{2}{3}$ if it is negative. We consider only the last possibility for the values enclosed in brackets of Table IV.1.

The parameters C_0 and C_1 are defined as follows

$$C_0 = \langle L=0 | \cos \theta(k) | L=0 \rangle = \sum_{\vec{k}} |M_0(\vec{k})|^2 \frac{m}{\sqrt{k^2 + m^2}}$$

$$C_1 = \langle L=1, L_z=+1 | \sqrt{2} \frac{k_+}{k} \sin \theta(k) | L=0 \rangle = \sum_{\vec{k}} |M_{1,1}(\vec{k})|^2 \frac{\sqrt{2} k_+}{\sqrt{k^2 + m^2}},$$

where $M_{L,L_z}(\vec{k})$ gives the k -distribution for a meson state with orbital angular momentum L, L_z . The results obtained with $C_0^2 = \frac{1}{2}$ and $C_1^2 = \frac{1}{3}$ are given in the third column of Table IV.1 and are to be compared with the experimental G^2 's in the fourth column, which come from the decay widths, as explained in section II.5.

It is worth noticing that so many decays can be well reproduced by a single operator like $Y^{\alpha 3,9}$!

Although C_0 and C_1 have been considered as parameters, we can try to understand their values from intuitive arguments⁹⁾. First let us take $m \approx \frac{1}{3}$ (proton mass): this value has been considered for years in the context of the quark model. The distribution $M_{L,L_z}(\vec{k})$ can be determined only for very specific models. But we can estimate the mean value $\langle k \rangle$ by noticing that strong interactions produce cross-sections which are very peaked around the low transverse momenta whose mean value is around $\frac{1}{3}$ of the proton mass. We then assume that for $L=0$ states, $\langle L=0 | k^2 | L=0 \rangle \approx m^2$; if we now take any reasonable distribution for $M_0(\vec{k})$ like a gaussian (as for the $n=0$ state of the harmonic oscillator) we get $C_0^2 \approx \frac{1}{2}$, which gives an excellent value for $G_{\rho\pi}$. The parameter C_1 is more difficult to estimate because of its stronger dependence on the k -distribution. But if one considers $n=0,1$ three-dimensional harmonic oscillator wave-functions, and

Table IV.1

G^2	CALCULATED VALUES	SAME WITH $C_0^2 = \frac{1}{2}$ $C_1^2 = \frac{1}{3}$	EXPERIMENTAL VALUES
$G_{p\pi}^2$	C_0^2	$\frac{1}{2}$	$.44 \pm .03$
$G_{w\pi}^2$	C_0^2	$\frac{1}{2}$	
$G_{A_2\rho}^2 (h=1)$	$\frac{1}{4} C_1^2$	$\frac{1}{12}$	$.065 \pm .005$
$G_{A_1\rho}^2 (h=0)$	C_1^2	$\frac{1}{3}$	$\left\{ \frac{G_{A_1\rho}^2(0)}{G_{A_1\rho}^2(1)} = 4^{+4}_{-1} \right.$
$G_{A_1\rho}^2 (h=1)$	$\frac{1}{4} C_1^2$	$\frac{1}{12}$	
$\sum_h G_{A_1\rho}^2 (h)$	$\frac{9}{4} C_1^2$	$\frac{3}{4}$	≈ 1
$G_{A_2\eta}^2$	$\frac{1}{9} C_1^2 \left[\frac{2}{27} C_1^2 \right]$	$\frac{1}{27} \left[\frac{2}{81} \right]$	$.023 \pm .004$
$G_{\delta\eta}^2$	$\frac{2}{9} C_1^2 \left[\frac{4}{27} C_1^2 \right]$	$\frac{2}{27} \left[\frac{4}{81} \right]$	$.08 \pm .05$
$G_{f\pi}^2$	$\frac{1}{3} C_1^2$	$\frac{1}{9}$	$.12 \pm .01$
$G_{E\pi}^2$	$\frac{2}{3} C_1^2$	$\frac{2}{9}$	$> .1$
$G_{B\omega}^2 (h=0)$	0	0	$\left\{ \frac{G_{B\omega}^2(0)}{G_{B\omega}^2(1)} = .05 - .5 \right.$
$G_{B\omega}^2 (h=1)$	$\frac{1}{2} C_1^2$	$\frac{1}{6}$	
$\sum_h G_{B\omega}^2 (h)$	C_1^2	$\frac{1}{3}$	$.40 \pm .07$

fixes the oscillator frequency from $C_0^2 \approx \frac{1}{2}$, i.e., $\langle L = 0 | k^2 | L = 0 \rangle \approx m^2$, we then get $C_1^2 \approx \frac{2}{5}$ which is quite near the value $\frac{1}{3}$ needed in order to fit the experimental decays! However, if we calculate in this harmonic oscillator model the matrix elements for $(L = 2) \rightarrow (L = 0)$ transitions, the results are about an order of magnitude smaller than what is suggested by one or two pieces of experimental information which are available.

We have already mentioned that some theoretical problems arise from the fact that $\tilde{V}_0^\alpha = V_0^\alpha$ (see section IV.1). But there are also some phenomenological reasons to look for modifications in our scheme: for example, the $B \rightarrow \omega\pi$ decay is predicted to be purely transverse, while experiment suggests some longitudinal part, i.e., some mixing for $\tilde{V}_0^\alpha$.

IV.4 ELECTROMAGNETIC TRANSITIONS

As another interesting application of our mixing operator, we can calculate the magnetic moment, which is defined as follows:

$$\vec{\mu} = e \int d^3x (\vec{x} \times \vec{J}^{EM}(x)). \quad (IV.15)$$

Notice that, if we make no distinction between current and constituent quarks, (i.e., in the absence of mixing) the latter would couple to $\vec{J}^{EM}(x)$ as the former do and, because of the $\vec{x}$ operator in Eq. (IV.15), the resulting $\vec{\mu}$ would have $L = 1$, and should vanish for $L = 0$ states. Instead, if we take the mixing transformation into account, $\vec{\mu}$ will have all L -values, in particular $L = 0$. The calculations are explicitly done in Appendix III; the results for $L = 0$ neutral mesons is ⁹⁾

$$\langle \tilde{M}_1^0 | \vec{\mu} | \tilde{M}_2^0 \rangle = \langle M_1^0 | \frac{e}{2m} \left(\frac{m^2 + \omega m}{2\omega^2} \right) \left(\frac{\vec{\sigma} \cdot \vec{\lambda}^{EM}}{2} + \frac{\vec{\sigma} \cdot \vec{\lambda}^{EM}}{2} \right) | M_2^0 \rangle, \quad (IV.16)$$

i.e., the naïve quark model expression (where $\vec{\mu}$ is given by the sum of the Dirac magnetic moments of the constituents) scaled down by a factor depending on the transverse kinetic energy of quarks. From Eq. (IV.16) and the states given in Appendix I, we calculate

$$\langle \pi^0 | \mu | \omega \rangle = - \frac{e}{2m} \langle L=0 | \frac{1 + \sqrt{1 + k^2/m^2}}{2(1 + k^2/m^2)} | L=0 \rangle$$

and under the same assumptions already made in the evaluation of the parameter C_0 in the preceding section, namely, a gaussian-like k -distribution for $L = 0$ states, with $\langle k \rangle_{L=0} \approx m \approx M_p/3$, we get ⁹⁾

$$\langle \pi^0 | \mu | \omega \rangle \approx -2 \frac{e}{2M_p}$$

to be compared to the experimental value -2.4 ± 0.3 , from the $\omega \rightarrow \pi^0 \gamma$ decay. In the same way we can calculate $\phi \rightarrow \eta \gamma$ and all other decays $V \rightarrow P \gamma$ are then obtained by SU(3) symmetry. The results are qualitatively good, but in order to get significant values, one has to take SU(3) breaking into account ⁶⁾.

CHAPTER V

BARYONS AS QUARK SYSTEMS

V.1 THE CHIRAL CONTENT OF BARYONS

In this chapter, the chiral content of baryons is studied along the same lines as those of the approach that we have considered in the case of meson states.

In order to apply hypotheses (A) and (B) of Chapter III, baryon states moving with $p_z \rightarrow \infty$ (and zero transverse momentum) are constructed as systems of three quarks, q_i ($i = 1, 2, 3$), each one with transverse momentum $\vec{k}_i$ and infinite momentum along the z-direction:

$$|\tilde{B}\rangle = |\tilde{B}_{p_z \rightarrow \infty}\rangle = \sum_{\vec{k}_i} B(\vec{k}_1, \vec{k}_2, \vec{k}_3) |\tilde{q}_1 \vec{k}_1\rangle |\tilde{q}_2 \vec{k}_2\rangle |\tilde{q}_3 \vec{k}_3\rangle, \quad (V.1)$$

where $B(\vec{k}_1, \vec{k}_2, \vec{k}_3) = B(\vec{k}_i)$ is some distribution in the quark transverse momenta. Because the composite baryon state is defined with zero transverse momentum, we have the condition

$$\vec{k}_1 + \vec{k}_2 + \vec{k}_3 = 0. \quad (V.2)$$

This trivial condition will play an important rôle in the forthcoming discussion. The baryon states are explicitly given in Appendix II as superpositions of three-quark states. From the point of view of the classification group, $SU(6)_W^{\text{class}}$, they belong to 56, 70 and 20 representations, as they are made of three constituent quarks. Because $SU(6)$ is not a symmetry, some mixing in the classification of states in representations of $SU(6)$ can, in principle, occur, as mentioned in section I.5.

In order to determine the chiral content of the baryon $|\tilde{B}\rangle$, we introduce "mathematical" states,

$$|B\rangle = \sum_{\vec{k}_i} B(\vec{k}_i) |q_1 \vec{k}_1\rangle |q_2 \vec{k}_2\rangle |q_3 \vec{k}_3\rangle \quad (V.3)$$

which transform simply as 56, 70, and 20 states under $SU(6)_W^{\text{curr}}$ operators, as they are made of three current quarks. Then, the mixing operator, defined in Eqs. (V.14) and (V.16) will give in the case of baryons:

$$|\tilde{B}\rangle = \sum_{\vec{k}_i} B(\vec{k}_i) \prod_{i=1}^3 U_i(\vec{k}_i) |q_i \vec{k}_i\rangle = e^{i \sum_i \Theta_i Z_i} |B\rangle \equiv e^{i \Theta Z} |B\rangle \quad (V.4)$$

$$\Theta_i \equiv \Theta(k_i) = \arctan \frac{k_i}{m} \quad Z_i = \frac{[\vec{\sigma}_i \times \vec{k}_i]_z}{2k_i}.$$

In this chapter, operators with an index i act only on the quark q^i . Eq. (V.4) gives the chiral content of baryons; for example, if $\tilde{B}$ is a proton with spin up, $|\tilde{p}\uparrow\rangle$, we shall find in general a large variety of states in the r.h.s., as $|p\uparrow\rangle$, $|p\downarrow\rangle$, $|p_3\uparrow\uparrow\rangle$, $|p_1\uparrow\rangle$, $|p_{20}\uparrow\rangle$, etc. (see Appendix II for the notation). Consequently, all of the chiral $SU(3) \times SU(3)$ representations $(\underline{6}, \underline{3})$, $(\underline{3}, \underline{6})$, $(\underline{\bar{3}}, \underline{3})$, $(\underline{3}, \underline{\bar{3}})$, $(\underline{8}, \underline{1})$ and $(\underline{1}, \underline{8})$ will appear in combinations that depend on the $B(\vec{k}_i)$ distribution. It is possible to show that, as a consequence of Eq. (V.4), the proton contains 3/2 more $(\underline{\bar{3}}, \underline{3})$ and $(\underline{3}, \underline{\bar{3}})$ than $(\underline{8}, \underline{1})$ and $(\underline{1}, \underline{8})$.

V.2 ALGEBRAIC PROPERTIES OF THE AXIAL CHARGE OPERATOR

For the current quark states $|B\rangle$ the axial charge operators are obtained from Eq. (III.17):

$$Q_5^\alpha |B\rangle = \sum_{\vec{k}_i} B(\vec{k}_i) \left(\sum_{i=1}^3 X_i^\alpha \right) \prod_{j=1}^3 |q_j \vec{k}_j\rangle \quad X_i^\alpha = \frac{1}{2} \lambda_i^\alpha \sigma_{iz} \quad (\alpha = 1, 2, \dots, 8) \quad (V.5)$$

By using Eq. (V.4) one gets for the axial charge between physical baryons,

$$\langle \tilde{B}' | Q_5^\alpha | \tilde{B} \rangle = \langle B' | e^{-i\theta Z} Q_5^\alpha e^{i\theta Z} | B \rangle . \quad (V.6)$$

Now, we define operators Y_i^α in the current quark base, as

$$y_i^\alpha = \frac{\lambda_i^\alpha \vec{\sigma}_i \cdot \vec{k}_i}{2k_i} \quad (i=1,2,3)$$

and notice the commutation rules

$$[Z_i, X_i^\alpha] = -i y_i^\alpha \quad [Z_i, Y_i^\alpha] = i X_i^\alpha . \quad (V.7)$$

With Eqs. (V.5)-(V.7) we can write the axial charge between physical states in the very simple form⁹⁾

$$\begin{aligned} \langle \tilde{B}' | Q_5^\alpha | \tilde{B} \rangle &= \langle B' | (X^\alpha - Y^\alpha) | B \rangle \\ X^\alpha &= \sum_i \cos \theta(k_i) X_i^\alpha \quad Y^\alpha = \sum_i \sin \theta(k_i) y_i^\alpha . \end{aligned} \quad (V.8)$$

All of the operators X^α , Y^α and θZ belong to 35 representations of $SU(6)_W^{\text{curr}}$. This is easily seen by considering their commutators with the $SU(6)_W^{\text{curr}}$ charges which, when acting on quarks with fixed values of $\vec{k}_i$, can be written as,

$$Q^\alpha = \frac{1}{2} \sum_i \lambda_i^\alpha , \quad Q_5^\alpha = \frac{1}{2} \sum_i \lambda_i^\alpha \sigma_{i_z} , \quad Q_\pm^\alpha = \frac{1}{2} \sum_i \lambda_i^\alpha \sigma_{i_\pm} . \quad (V.9)$$

In particular, if we consider only the chiral subgroup, $SU(3) \times SU(3) \subset SU(6)_W^{\text{curr}}$, we find that the X^α 's belong to a $(\underline{8}, \underline{1}) + (\underline{1}, \underline{8})$ representation, Y^α to a $(\underline{3}, \underline{\bar{3}}) + (\underline{\bar{3}}, \underline{3})$ one; θZ behaves also as a $(\underline{3}, \underline{\bar{3}}) + (\underline{\bar{3}}, \underline{3})$ operator (but, in general, it is not in the same multiplet that contains the Y^α). Notice however, that $e^{i\theta Z}$ is not a 35 as $(\theta Z)^N$ can have components in many different representations which connect one three-quark state to another and

are contained in (35)^N. Therefore, the chiral content can be very involved, even if the charges are simple. For this reason we prefer to discuss the physical consequences of our hypotheses in terms of the operators θZ , X^α and Y^α . It has been shown in a previous publication⁹⁾ that they are intimately related to the model proposed in Ref. 3, but we shall not insist very much on this point here and we shall rather concentrate on the consequences of Eq. (V.8).

Besides being 35 operators, X^α , Y^α and θZ have some definite properties under rotations, once the "total quark spin" $\vec{S}$, and the "orbital angular momentum of the quark system", $\vec{L}$, are defined. The operators θZ and Y^α have only $S = 1$, $S_z = -L_z = \pm 1$ parts, with all odd values for L . The operator X^α has $S = 0$ and $S = 1$ parts, $S_z = L_z = 0$ and all even values for L . These properties under spatial rotations are easily seen from the fact that the $\vec{k}_i$ are vectors under $O(3)_L$ and commute amongst themselves, so that $\cos\theta(k_i)$ and $\sin\theta(k_i)/k_i$ can be developed in powers like $(k_{i+}k_{i-})$, which has $L = 2n, 2n-2, \dots, 0$; the additional $k_{i\pm}$ in Y^α and Z shifts the L values by one unit.

V.3 CONSEQUENCES FOR THE BARYON SPECTRUM

Now, if we use PCAC to relate the matrix elements of Q_5^α to pionic couplings, we can discuss which are the states that are allowed in our model to decay into a nucleon (or another member of the stable baryon octet) plus one pion. Remember that nucleons are classified in a 56 $L = 0$ representation. The axial charge (and so the pion) couple the nucleons to higher even L states through the X^α operators and to odd states through the Y^α ones.

Both X^α and Y^α are combinations of the two possible types of 35 operators: one which couples a 56 only to a 56, another which couples a 56 to a 70 only.

We shall give some examples⁹⁾ in order to illustrate the connection between the baryon $SU(6)$ -spectrum and the spatial configuration, i.e. the distribution of the relative momenta of quarks; it arises in our model from the fact that the chiral content of each quark inside the baryon depends on its transverse momentum.

Let us introduce the notation $A|\underline{R}\rangle = |\underline{R}'\rangle + |\underline{R}''\rangle$ to mean that the operator A brings a state of the representation $\underline{R}$ to others in the representations $\underline{R}'$ and $\underline{R}''$ of $SU(6)$. Now, it is easy to see that Y^α cannot satisfy $Y|\underline{56}\rangle = |\underline{56}\rangle$; indeed, this would imply Y^α to be proportional to a combination of $SU(6)$ generators [the $Q_\pm^\alpha$ in Eq. (V.9)] and so $\vec{k}_1 = \vec{k}_2 = \vec{k}_3$, in contradiction with Eq. (V.2). Thus, a spectrum of only 56 states is excluded, and some 70 ones are needed, at least for odd L 's (in agreement with the good experimental evidence for states in a 70 ($L = 1$)). Instead, we might have only 56 states for even L 's, if $|\vec{k}_1| = |\vec{k}_2| = |\vec{k}_3|$, so that X^α would be proportional to the operator Q_5^α in Eq. (V.9); otherwise $X|\underline{56}\rangle = |\underline{56}\rangle + |\underline{70}\rangle$.

As a more specific example, consider baryon states such that $k_1 = k_2 = k_3$, with azimuthal angles $\phi_1 = \phi_2 - 2\pi/3 = \phi_3 - 4\pi/3$; in this case $X^\alpha|\underline{56}\rangle = |\underline{56}\rangle$ and $Y^\alpha|\underline{56}\rangle = |\underline{70}\rangle$ (also $\theta Z|\underline{56}\rangle = |\underline{70}\rangle$), in such a way that pion-nucleon resonances should all be in 70 representations for odd L and in 56 ones for even L . This alternate spectrum has been popular for some time but now it has fallen into disfavour¹⁾. Another example: take baryon states with $\vec{k}_1 = 0$, $\vec{k}_2 = -\vec{k}_3$; then $Y|\underline{56}\rangle = |\underline{70}\rangle$ and $\theta Z|\underline{56}\rangle = |\underline{70}\rangle$, but $X|\underline{56}\rangle = |\underline{56}\rangle + |\underline{70}\rangle$.

In general, all of the operators X^α , Y^α and θZ bring a 56 state into 56 and 70 ones; because they are 35 operators they cannot couple the nucleon (in a 56) to baryons belonging to a 20 representation. This gives a selection rule: 20 baryons cannot go into a nucleon plus one pion, their simplest decay mode being into a nucleon plus two pions (through 70 baryons plus one pion) as long as the charge is made of 35 operators, which is the case in the present model.

The restrictions become even stronger whenever one imposes definite statistics to quarks, implying baryon states with a given permutational symmetry in the three quarks. Indeed, the operators which we discuss here, X^α , Y^α , θZ , are all symmetric in the three quarks. This means that they keep the symmetry of the states under permutations: once it is fixed for the nucleon, all the resonances must be of the same permutational type. In other words, our operators are consistent with definite

quark statistics. This property can be traced back to Eq. (V.2), i.e., to the fact that we consider only hadron states with zero transverse momentum.

V.4 AXIAL CHARGES IN THE SYMMETRIC QUARK MODEL

As already mentioned in section I.4, baryons seem to be symmetric states of three constituent quarks. In Appendix II a simple approach to the symmetrization of three-particle states is given: for the assignment of a given quantum number to quarks, we have four types of permutational configuration: S (symmetric), α and β (mixed symmetry) and A (antisymmetric); in order to symmetrize with respect to two independent sets of quantum numbers we have three possibilities:

$$|x_1 x_2 x_3\rangle_S |y_1 y_2 y_3\rangle_S ; |x_1 x_2 x_3\rangle_\alpha |y_1 y_2 y_3\rangle_\alpha + |x_1 x_2 x_3\rangle_\beta |y_1 y_2 y_3\rangle_\beta ; |x_1 x_2 x_3\rangle_A |y_1 y_2 y_3\rangle_A .$$

We see that, as 56 states are symmetric in the SU(6) indices, the spatial configuration of the quarks must be symmetric: baryons in a 56 can be represented by $|56\rangle_S |space\rangle_S$. The 70 representations of SU(6) have mixed symmetry, so that symmetric baryon states in a 70 must be of the form $|70\rangle_\alpha |space\rangle_\alpha + |70\rangle_\beta |space\rangle_\beta$. Finally, the [SU(6)-antisymmetric] 20 baryons should be $|20\rangle_A |space\rangle_A$.

Hereafter, we assume the nucleon states in a 56, $L = 0$ to be symmetric in the three quarks; our operators imply that all other baryons must be so. Thus, from the discussion in the preceding section, we see that in our model nucleons are coupled only to the states: $|56\rangle_S |space\rangle_S$ and $(|70\rangle_\alpha |space\rangle_\alpha + |70\rangle_\beta |space\rangle_\beta)$. As the X^α and Y^α are explicitly given in Eq. (V.8), we can immediately calculate the SU(6) part of the axial charge matrix elements. It is worth while noticing that for calculational purposes we can replace $X^\alpha = \sum_i \cos\theta(k_i) X_i$ by $3X_1 \cos\theta(k_1)$ for example, as long as it is sandwiched between symmetric states; and analogously for Y^α .

The resulting matrix elements are given in Table V.1 for the non-strange states classified in $SU(6) \times O(3)_L$. The notation for SU(6) states is defined in Appendix II; the spatial configuration is labelled by L, L_z , and the index n, at this stage, is to be interpreted as representing

additional quantum numbers needed in order to distinguish different sets of states with the same $SU(6) \times O(3)$ classification; also from Eq. (V.4)

$$\cos \theta(k_1) = \frac{m}{\sqrt{k_1^2 + m^2}} \quad \sin \theta(k_1) = \frac{k_1}{\sqrt{k_1^2 + m^2}}. \quad (V.10)$$

If we take fully into account the $SU(3)$ symmetry (which is not broken in our approximate scheme) we get the results given in Table V.2. The coefficients F , D and C are defined in Appendix II. The additional information here with respect to Table V.1 are the F/D ratios (i.e., the coefficients of the F 's in the table) of the axial charges between excited octet states and the stable baryon octet, as well as the matrix elements of the $SU(3)$ -singlet members of the 70-plets.

From these matrix elements we can calculate, by using PCAC, the pionic couplings of baryon resonances: all we have to do is to project our $|S S_z, L L_z\rangle$ states into $|L S, J J_z\rangle$ ones and identify the latter with experimental resonances (see Chapter I). For each given representation of $SU(6) \times O(3)$ one has only one parameter which for the moment is free. In Table V.3) we give the predictions for the pionic couplings of the well-established resonances and their experimental values, obtained through the formalism of section II.4, from the decay rates³⁾. For each $SU(6) \times O(3)$ multiplet we choose the better known resonance in order to determine the free parameter in the matrix elements. The G 's are related to the axial charge matrix elements as follows

$$G_{N\alpha} = 2 \langle p \uparrow | Q_S^3 | \alpha \uparrow \rangle \quad \text{if } I_\alpha = \frac{1}{2}$$

$$G_{N\alpha} = \sqrt{2} \langle p \uparrow | Q_S^3 | \alpha \uparrow \rangle \quad \text{if } I_\alpha = \frac{3}{2}.$$

Our results are qualitatively good, and many of the discrepancies could be explained by the fact that we have some mixing in the $SU(6) \times O(3)$ classification, as already mentioned in section I.5. The experimental errors are not given and often they are quite large¹⁴⁾. Anyway,

we should remark that the number of parameters is very small compared to the large number of predictions. Notice that we can also predict the phases of the matrix elements; recently, they have been experimentally determined by phase-shift analyses and agree quite well with our results²⁴⁾.

The more interesting feature of the model is the prediction for the axial charge between the basic 56 $L = 0$ states. If we define

$$\langle p\uparrow | Q_5^3 | p\uparrow \rangle = \frac{1}{2} G_A \quad \langle p\uparrow | Q_5^3 | \Delta\uparrow \rangle = \frac{1}{\sqrt{2}} G^* \quad (V.11)$$

$$\langle 8\uparrow | Q_5^\alpha | 8\uparrow \rangle = G_A (D^\alpha + f F^\alpha)$$

we get $G^* = 4/5 G_A$ and $f = 2/3$, i.e. these SU(6) results are conserved by the mixing operator, in very good agreement with experiments. This follows simply from the fact that our axial charges contain only 35 operators, and not from their specific form. The agreement can be considered as experimental evidence for this hypothesis.

V.5 A HARMONIC OSCILLATOR MODEL FOR BARYONS

In order to have an explicit model where the matrix elements can be completely calculated, we consider here the symmetric harmonic oscillator quark model¹⁾. There are at least four reasons which have been given as motivations for such a choice:

- i) all the states being bound, we need not worry about the existence of free quarks and of a free-quark mass;
- ii) the energy spectrum of the states should rise linearly, as Regge trajectories seem to do;
- iii) in the Veneziano model we have an infinite number of harmonic oscillators corresponding to a fundamental mode and all its harmonics; perhaps, as a first approximation for the dynamics of low-lying states, we might consider only the first mode;

iv) the predicted pattern for the $SU(6) \times O(3)$ spectrum is consistent with the observed resonances (n is the principle quantum number):

H.O. model	Experimental evidence
n = 0 : $\underline{56}$ L = 0	very good
n = 1 : $\underline{70}$ L = 1	good
n = 2 : $\left\{ \begin{array}{l} \underline{56} \text{ L} = 2 \\ \underline{56} \text{ L} = 0 \\ \underline{70} \text{ L} = 2 \\ \underline{70} \text{ L} = 0 \\ \underline{20} \text{ L} = 1 \end{array} \right.$	good
	some
	some
	none
	some

In this model, the three quarks are coupled together by harmonic forces. We define H.O. creation and annihilation operators, a_i^* and a_i ($i = 1, 2, 3$), associated to the relative position and momentum operators of each quark. They are not independent, and can be expressed in terms of two linear combinations

$$a = \frac{1}{\sqrt{6}} (2a_1 - a_2 - a_3) \quad b = \frac{1}{\sqrt{2}} (a_2 - a_3). \quad (V.12)$$

If now we define

$$\vec{\xi} = \frac{1}{\sqrt{2}} (\vec{a} + \vec{a}^*) \quad \vec{\eta} = \frac{1}{\sqrt{2}} (\vec{b} + \vec{b}^*) \quad (V.13)$$

we can choose the unit of mass in such a way that the quark momenta are expressed as follows:

$$\vec{k}_1 = \sqrt{\frac{2}{3}} \vec{\xi} \quad \vec{k}_2 = -\frac{1}{\sqrt{6}} \vec{\xi} + \frac{1}{\sqrt{2}} \vec{\eta} \quad \vec{k}_3 = -\frac{1}{\sqrt{6}} \vec{\xi} - \frac{1}{\sqrt{2}} \vec{\eta} \quad (V.14)$$

and they satisfy Eq. (V.2). Then, the principal quantum number, n , and the orbital angular momentum operator $\vec{L}$, are defined in the usual way. The index n in the tables is to be interpreted now as the principal quantum number.

It is important to notice that the operators $\vec{a}^*$ ($\vec{b}^*$) create a state of the α - (β -) type, as a consequence of their definitions in Eq. (V.12). Then the construction of the states and the calculation of the matrix elements in the tables are straightforward¹¹⁾.

The H.O. states are to be combined symmetrically with the SU(6) states. The number of different possibilities (each one corresponding to physical states!) increase very rapidly with n . Here we shall only look at some specific consequences of this spectrum related to the experimental information available for pionic couplings.

By looking at Tables V.1 and V.2 we see that all we need to know are the components of each state which have only excitations by the $a_{\pm}^*$ operators and no a_z^* or $\vec{b}^*$ excitation. They are:

- (1) of the type S, associated to 56 states;
- (2) of the type α , associated to the 70 states.

Thus we can calculate the amount of 56 and 70 states which are coupled to the nucleons for a given value of n if we determine the ratio $\alpha/S)_n$, i.e., the relative amount of α and S parts in the states of the type

$$(a_+^* a_-^*)^n |0\rangle \quad \text{and} \quad a_{\pm}^* (a_+^* a_-^*)^n |0\rangle .$$

This is easily done by constructing a recurrence relation, and the result is

$$\left(\frac{\alpha}{S} \right)_n = \frac{1 - \left(-\frac{1}{2}\right)^n}{\frac{1}{2} + \left(-\frac{1}{2}\right)^n} . \quad (V.15)$$

This ratio will be useful in the next section. Notice that for large n , $\alpha/S \rightarrow 2$, and that this value is approached very quickly, in an oscillatory way.

For $n = 2$, $\alpha/S)_2 = 1$; if we put this ratio in the matrix elements of Table V.1, we obtain¹¹⁾

$$\frac{\sum_{L=0}^2 |\langle p\uparrow | Q_5^3 | p\uparrow \rangle_{56} | L, L_z=0 \rangle_{\text{symm.}}^{n=2}|^2}{\sum_{L=0}^2 \sum_{\gamma=1,3} |\langle p\uparrow | Q_5^3 | p\uparrow \rangle_{\gamma 0} | L, L_z=0 \rangle_{\text{symm.}}^{n=2}|^2} = \frac{5}{2} \quad (V.16)$$

$$\frac{\sum_{L=0}^2 |\langle p\uparrow | Q_5^3 | \Delta\uparrow \rangle_{56} | L, L_z=0 \rangle_{\text{symm.}}^{n=2}|^2}{\sum_{L=0}^2 |\langle p\uparrow | Q_5^3 | \Delta\uparrow \rangle_{70} | L, L_z=0 \rangle_{\text{symm.}}^{n=2}|^2} = 16 .$$

Therefore, we predict that 56 states with $n = 2$ are coupled much more to nucleons than the 70 ones. This could explain the fact that until a few months ago, only the states to be classified in the 56 $L = 2$ and 56 $L = 0$ were known for $n = 2$.

Also, if we project the state $a_+^* a_-^* |0\rangle$ into its $L = 2$ and $L = 0$ parts, we obtain¹¹⁾

$$\frac{|\langle p\uparrow | Q_5^3 | p\uparrow \rangle_{56} | 00 \rangle^{n=2}|^2}{|\langle p\uparrow | Q_5^3 | p\uparrow \rangle_{56} | 20 \rangle^{n=2}|^2} = 2 \quad (V.17)$$

to be compared to the experimental value 1.3 ± 0.8 .

We can easily calculate all the matrix elements,

$$\langle 0 | \cos \theta(k_1) | L 0 \rangle_{S, \alpha}^n \quad \langle 0 | \frac{k_{1+}}{k_1} \sin \theta(k_1) | L, -1 \rangle_{S, \alpha}^n$$

which appear in Tables V.1, V.2, and V.3. For example, by using Eq. (V.10) we get:

$$\langle 0 | \cos \theta(k_1) | L 0 \rangle_{\alpha}^n = \frac{2}{3} \left(1 - \frac{1}{2^n}\right) \frac{\Gamma(n+\frac{1}{2})}{\sqrt{\pi m}} \Psi\left(n+\frac{1}{2}, \frac{1}{2}; \frac{1}{m}\right). \quad (V.18)$$

This expression, with its confluent hypergeometric function, is not very transparent. Rather, we shall consider some simple cases. As we have already done for mesons, and for the same reasons given there, we fix the quark mass by imposing $\langle k_i^2/m^2 \rangle_{L=0} = 1$ ($i = 1, 2, 3$) for the mean value of the quark transverse momenta in the fundamental state. We then find

$$\langle n = 0 | \cos \theta(k_1) | n = 0 \rangle = 0.757$$

which gives for the G_A constant, defined in Eq. (V.11), $G_A \approx 1.26$, in good agreement with the experimental value.

Now that all the parameters are fixed, we also determine

$$\langle 0 | \sqrt{2} \frac{k_+}{k_1} \sin \theta(k_1) | 1, -1 \rangle_{\alpha}^{n=1} \approx 0.61$$

giving the value $G^2 \approx 1.1$ for the $N(1520)$ resonance, to be compared with the experimental one, $G^2 \approx 0.9$, as quoted in Table V.3. The relative couplings inside the 7C $n = 1$, $L = 1$, are obviously the same as given in this table.

However, if we calculate the matrix elements for $n = 2$, they turn out to be an order of magnitude too small to account for the experimental couplings of the 56 $L = 2$ and 56 $L = 0$ given in Table V.3. We conclude that our scheme needs some modifications in order to reduce the couplings of the $n = 1$ states and increase those of $n = 2$ ones. We have already found the same problem in the case of mesons (section IV.3). Because the relative strengths at fixed n are quite reasonable, we might think that Eq. (V.10) is to be changed, but in this case we would lose the nice physical interpretation of $\theta(k)$ as the angle of the Melosh transformation. This point is under investigation now and it is discussed in more detail in Ref. 11.

V.6 JOHNSON-TREIMAN RELATIONS; C.H.N. PATTERN IN THE CROSSED CHANNEL

We have seen that the model presented here for the chiral content of baryons gives a good over-all description of pionic coupling constants.

Therefore, we can determine the resonant part of the meson baryon amplitudes once the masses of the intermediate states are known. The empirical hypothesis of duality¹⁹⁾ says that the averaged contribution of resonances to the s-channel of a given reaction corresponds to the exchange of Regge poles in the crossed t-channel. Here, we show that the t-channel properties of the meson-baryon amplitudes resulting from our couplings have an interesting, very realistic structure. Indeed, the Johnson-Treiman relations¹³⁾ are satisfied everywhere (in an energy averaged sense), and for higher energies the t-channel amplitudes follow the pattern proposed by Cabibbo, Horwitz and Ne'eman¹³⁾ on the basis of an algebra for the couplings of Regge trajectories¹¹⁾. In particular, the exotic exchanges are suppressed at high energies. [A state is called exotic if, from the point of view of SU(6), it cannot be classified in a 1-plet or a 35-plet for mesons, a 20-plet, a 56-plet or a 70-plet for baryons.]

In order to show that we calculate the contribution of each $SU(6) \times O(3)$ supermultiplet to the absorptive part of the forward amplitude for the process $PB \rightarrow PB$, where P is the octet of pseudoscalar mesons (π , K, etc.) and B represents the octet of stable baryons (p, Σ , etc.). This reaction is of the SU(3)-type $\underline{8} + \underline{8} \rightarrow \underline{8} + \underline{8}$; but $\underline{8} \times \underline{8} = \underline{1} + \underline{8}_a + \underline{8}_s + \underline{10} + \underline{10} + \underline{27}$, so that there are seven independent s-channel amplitudes in the limit of exact SU(3): $A_1^s, A_{8a8a}^s, A_{8s8s}^s, A_{8a8s}^s = A_{8s8a}^s, A_{10}^s, A_{\underline{10}}^s$ and A_{27}^s .

Here, we are interested in the "semilocal dual" properties, i.e., those following from an average of the imaginary part of the amplitudes on a small-range energy interval, so that we can take for the mass of all the states in a given $SU(6) \times O(3)$ supermultiplet the same value, which we need not specify for our purposes. Also, in the limit of exact SU(3) symmetry, the PCAC relation between pionic couplings and non-strange axial charges is extended to the couplings of the whole octet of pseudo-scalar mesons, which are then proportional to the corresponding axial charge matrix elements. From the global contributions of octets, decuplets and singlets in a supermultiplet, as given in Table V.2, we can write for the absorptive part of each of the seven amplitudes above, in the energy

region (labelled by n) where they are dominated by one of these sets of resonances:

$$A_1^s = 6 \sigma_n \beta_n \quad A_{8_1 8_a}^s = \left(\frac{4}{3} \rho_n + \frac{29}{12} \sigma_n \right) \beta_n$$

$$A_{8_a 8_s}^s = A_{8_s 8_a}^s = \left(\frac{4}{3} \rho_n + \frac{1}{6} \sigma_n \right) \sqrt{5} \beta_n \quad (V.19)$$

$$A_{8_s 8_s}^s = \left(\frac{5}{3} \rho_n + \frac{25}{12} \sigma_n \right) \beta_n \quad A_{10}^s = \left(\frac{8}{3} \rho_n + \frac{1}{3} \sigma_n \right) \beta_n ,$$

where ρ_n, σ_n are defined in Table V.2 and β_n is a constant for each supermultiplet n . Eq. (V.19) is true for any n . Notice that $A_{10}^s = A_{27}^s = 0$ because there are no exotic resonances in the s -channel. The crossed t -channel amplitudes correspond to the processes $P\bar{P} \rightarrow \bar{B}B$, which is also of the type $\underline{8} + \underline{8} \rightarrow \underline{8} + \underline{8}$, and they can be determined by applying the SU(3) crossing matrix on the amplitudes in Eq. (V.19). One obtains

$$A_1^t = \left(19 \rho_n + 17 \sigma_n \right) \frac{\beta_n}{3} \quad A_{8_s 8_s}^t = -\frac{7}{6} (\rho_n - \sigma_n) \beta_n$$

$$A_{8_s 8_a}^t = -\frac{\sqrt{5}}{6} (8 \rho_n + \sigma_n) \beta_n \quad A_{8_a 8_a}^t = \frac{3}{2} (\rho_n + 2 \sigma_n) \beta_n \quad (V.20)$$

$$A_{27}^t = -\frac{1}{3} (\rho_n - \sigma_n) \beta_n \quad A_{8_a 8_s}^t = A_{10}^t = A_{\bar{10}}^t = 0 .$$

When we consider the difference $A(PB) - A(\bar{P}\bar{B})$ only the amplitudes $A_{8_a 8_a}^t$, $(A_{10}^t - A_{\bar{10}}^t)$ and $A_{8_a 8_s}^t$ could contribute. The latter two vanish in our model, we have only one amplitude in this case, and we get the relations:

$$A(K^-p) - A(K^+p) = 2[A(\pi^-p) - A(\pi^+p)] = 2[A(K^0p) - A(K^0p)] =$$

$$= (\rho_n + 2 \sigma_n) \beta_n$$

which, by using the optical theorem, gives the Johnson-Treiman relations:

$$\begin{aligned} \frac{1}{2} \{ \sigma_T(K^-p) - \sigma_T(K^+p) \} &= \sigma_T(\pi^-p) - \sigma_T(\pi^+p) = \\ &= \sigma_T(\bar{K}^0p) - \sigma_T(K^0p). \end{aligned} \quad (V.21)$$

These equations are well verified experimentally at all energies. They have been derived here without specifying either the spatial structure of the states or the relative importance of 56 and 70 resonances. The only ingredient was the fact that the axial charges are 35 operators, and Eq. (V.20) can be looked at as a test of this SU(6) structure of charges.

Now, we assume the symmetric H.O. quark model and look for the crossed-channel amplitude when the principal quantum number n is large; we take the same mass for all the SU(6) $\times$ O(3) supermultiplets at a given value of n : From Eq. (V.15), $\alpha/S_n \rightarrow 2$ so that $\rho_n/\sigma_n \rightarrow 1$ for $n \rightarrow \infty$, and the t -channel amplitudes become:

$$A_1^t = 12 \rho_n \beta_n \quad A_{8_a 8_a}^t = \frac{9}{2} \rho_n \beta_n \quad A_{8_s 8_a}^t = -\frac{3\sqrt{5}}{2} \rho_n \beta_n \quad (V.22)$$

$$A_{8_s 8_s}^t = A_{8_a 8_s}^t = A_{10}^t = A_{\bar{10}}^t = A_{27}^t = 0.$$

There is no $\bar{B}B$ coupling of the D-type in the t -channel, because $A_{8_s 8_s}^t = A_{8_a 8_s}^t = 0$, and $A_{10}^t = A_{\bar{10}}^t = A_{27}^t = 0$ implies the absence of exotic exchanges! With the help of the Clebsch-Gordan coefficients in Appendix II, the whole t -channel amplitude can be written in the simple form¹¹⁾

$$\begin{aligned} \langle P^\alpha B^\gamma | A | P^\beta B^\delta \rangle &= \\ &= \frac{3}{2} \rho_n \beta_n \left\{ \delta_{\alpha\beta} \delta_{\gamma\delta} + \sum_{\nu} (F_{\alpha\beta}^{\nu} - D_{\alpha\beta}^{\nu}) F_{\gamma\delta}^{\nu} \right\} \end{aligned} \quad (V.23)$$

which corresponds to [the SU(3)-limit of] the pattern proposed by Cabibbo, Horwitz and Ne'eman for the t -channel Regge exchanges, namely:

- i) exchange degeneracy (EXD) for vector and tensor trajectories;
- ii) pure F-coupling of tensor and vector trajectories to baryons.

Experimentally, EXD is a good first approximation and the D/F ratio of the coupling between Regge trajectories and baryons is ≈ -0.2 .

Notice that Eqs. (V.22) and (V.23) are exact only in the limit of $n \rightarrow \infty$. But they are very good approximations as σ_n/ρ_n approaches the asymptotic value, 1, very quickly:

n =	0	1	2	3	4	5	6	7
$\sigma_n/\rho_n =$	0	∞	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{6}$	$\frac{11}{10}$	$\frac{21}{22}$	$\frac{43}{42}$

This alternate series implies small oscillations in some differences between total cross-sections; however the present experimental data are not accurate enough to look for such an effect there.

We remark that these properties are consequences of the SU(6) pattern chosen by the symmetric H.O. quark model; as they are only asymptotically true, they cannot follow merely from the algebraic features of the axial charges, as the Johnson-Treiman relations do.

Table V.1

MATRIX ELEMENTS OF Q_5^3

$ BL\uparrow\rangle_{\text{Symm.}}^n$	$\langle p\uparrow Q_5^3 BL\uparrow\rangle_{\text{Symm.}}^n$
$ p\uparrow\rangle_{56} \text{EVEN } L, L_z = 0 \rangle^n$	$\frac{5}{6} S_n^L$
$ \Delta\uparrow\rangle_{56} \text{EVEN } L, L_z = 0 \rangle^n$	$\frac{4}{3\sqrt{2}} S_n^L$
$ p_3\uparrow\rangle_{70} \text{EVEN } L, L_z = 0 \rangle^n$	$\frac{1}{3\sqrt{2}} \alpha_n^L$
$ p_1\uparrow\rangle_{70} \text{EVEN } L, L_z = 0 \rangle^n$	$-\frac{\sqrt{2}}{3} \alpha_n^L$
$ \Delta_1\uparrow\rangle_{70} \text{EVEN } L, L_z = 0 \rangle^n$	$\frac{1}{3\sqrt{2}} \alpha_n^L$
$ p\downarrow\rangle_{56} \text{ODD } L, L_z = +1 \rangle^n$	$\frac{5}{6} S_n^L$
$ \Delta\downarrow\rangle_{56} \text{ODD } L, L_z = +1 \rangle^n$	$\frac{\sqrt{2}}{3} S_n^L$
$ \Delta\uparrow\rangle_{56} \text{ODD } L, L_z = -1 \rangle^n$	$-\sqrt{\frac{2}{3}} S_n^L$
$ p_3\downarrow\rangle_{70} \text{ODD } L, L_z = +1 \rangle^n$	$\frac{1}{6\sqrt{2}} \alpha_n^L$
$ p_1\downarrow\rangle_{70} \text{ODD } L, L_z = +1 \rangle^n$	$-\frac{\sqrt{2}}{3} \alpha_n^L$
$ \Delta_1\downarrow\rangle_{70} \text{ODD } L, L_z = +1 \rangle^n$	$\frac{1}{3\sqrt{2}} \alpha_n^L$
$ p_3\uparrow\rangle_{70} \text{ODD } L, L_z = -1 \rangle^n$	$-\frac{1}{2\sqrt{6}} \alpha_n^L$

$[S_n^L, \alpha_n^L \text{ DEFINED IN TABLE V.2}]$

CONCLUDING REMARKS

We have seen that the model presented here has many advantages:

- i) it is very simple and presents the same algebraic structure found by Melosh in the free-quark model;
- ii) it gives a good qualitative description of axial charge matrix elements for all resonances;
- iii) the resulting meson-baryon amplitudes have the right t-channel properties, and in this sense the model is a realization of algebraic duality;
- iv) even the electromagnetic transitions can be understood in this quark model.

We have also considered a harmonic oscillator model for the spatial configurations. When the oscillator frequency is fixed (notice that it turns out to be the same for both mesons and baryons), the resulting couplings at the $n = 1$ level are roughly 15% too high (for both mesons and baryons) while the $n \geq 2$ couplings are too small. This effect seems to be mostly due to the particular form of $\theta(k)$ in the Melosh transformation. Had we chosen $\theta(k) = k/m$, as in the model of Ref. 3, the results would be much better from this point of view. With the phenomenological fact in mind, we could look for a justification of a modified form for $\theta(k)$.

But the important question which has to be answered is: what is the rôle of the interactions between quarks? in which extent is it responsible for the chiral content of hadrons? In this work we have supposed that, as argued in Ref. 23, interactions are only responsible for the observed breaking of $SU(6)_W$ in Nature, the chiral content being due to the kinetic energy of the quarks inside hadrons. The phenomenological successes of our model indicates that this is a good assumption, at least as a first approximation. The next step should be to justify this fact from a theoretical point of view.

APPENDIX I

NOTATIONS AND CLASSIFICATION OF THE MESON STATES IN $SU(6) \times O(3)$

In Tables A.I.1-A.I.3 we give some notations and results that are used in the text. Their contents are as follows:

Table A.I.1

Decomposition of the states of a $\underline{35} + \underline{1}$ into representations of $SU(3) \times SU(2)_S \subset SU(6)$, of $SU(3) \times SU(2)_W \subset SU(6)_W$ and the chiral $SU(3) \times SU(3) \subset SU(6)$ or $SU(6)_W$. As usual, the representations are labelled by their dimension. The non-vanishing matrix elements of the axial charges are also given there; they can be inferred from the matrix elements of the axial charges for the representations of the chiral algebra in Table A.II.3.

Table A.I.2

Realization of non-strange $\underline{35} + \underline{1}$ states in terms of $\bar{q}q$ pairs. These states are labelled by the name of the corresponding ($L = 0$) meson, with an upper index for the charge and a lower index for the S_z value (when needed).

Table A.I.3

$SU(6) \times O(3)$ states for the non-strange ($L = 1$) mesons. They are denoted by the name of the corresponding particle in Ref. 14, where the masses and the decay widths used in Chapter IV came from.

Table A.I.1

THE DECOMPOSITION OF THE $35+1$

SU(6) STATE	SU(3) x SU(2)				SU(3) x SU(3)
	SU(3)	S	W	$S_2=W_2$	(CHIRAL)
$V_{\pm 1}^{\alpha} (\alpha=1\dots 8)$	<u>8</u>	1	1	± 1	$\frac{1}{\sqrt{2}}[(\underline{3}, \underline{\bar{3}}) - (\underline{\bar{3}}, \underline{3})]_{\pm}$
$V_{\pm 1}^0$	<u>1</u>	1	1	± 1	$\frac{1}{\sqrt{2}}[(\underline{3}, \underline{\bar{3}}) - (\underline{\bar{3}}, \underline{3})]_{\pm}$
$V_0^{\alpha} (\alpha=1\dots 8)$	<u>8</u>	1	0	0	$\frac{1}{\sqrt{2}}[(8, 1) - (1, 8)]$
$P^{\alpha} (\alpha=1\dots 8)$	<u>8</u>	0	1	0	$\frac{1}{\sqrt{2}}[(8, 1) + (1, 8)]$
P^0	<u>1</u>	0	1	0	(1, 1)
V_0^0	<u>1</u>	<u>1</u>	0	0	(1, 1)'

AXIAL CHARGE MATRIX ELEMENTS

$$\langle P^{\beta} | Q_5^{\alpha} | V_0^{\gamma} \rangle = F^{\alpha\beta\gamma}$$

$$\langle V_{\pm 1}^{\beta} | Q_5^{\alpha} | V_{\pm}^{\gamma} \rangle = D^{\alpha\beta\gamma}$$

$$\langle V_{\pm}^{\beta} | Q_5^{\alpha} | V_{\pm}^0 \rangle = \sqrt{\frac{2}{3}} \delta^{\alpha\beta}$$

F, D DEFINED IN TABLE A.II.3

Table A.I.2

99 STATES OF MESONS IN A $35+1$ (ONLY $Y=0$) STATES

$$\rho_{+1}^+ = p \uparrow \bar{n} \uparrow \quad \rho_{+1}^- = n \uparrow \bar{p} \uparrow \quad \rho_{+1}^0 = \frac{1}{\sqrt{2}} (p \uparrow \bar{p} \uparrow + n \uparrow \bar{n} \uparrow)$$

$$\rho_{-1}^+ = p \downarrow \bar{n} \downarrow \quad \rho_{-1}^- = n \downarrow \bar{p} \downarrow \quad \rho_{-1}^0 = \frac{1}{\sqrt{2}} (p \downarrow \bar{p} \downarrow + n \downarrow \bar{n} \downarrow)$$

$$\rho_0^+ = \frac{1}{\sqrt{2}} (p \uparrow \bar{n} \downarrow + p \downarrow \bar{n} \uparrow) \quad \rho_0^- = \frac{1}{\sqrt{2}} (n \uparrow \bar{p} \downarrow + n \downarrow \bar{p} \uparrow)$$

$$\rho_0^0 = \frac{1}{2} (p \uparrow \bar{p} \downarrow + p \downarrow \bar{p} \uparrow + n \uparrow \bar{n} \downarrow + n \downarrow \bar{n} \uparrow)$$

$$\pi^+ = \frac{1}{\sqrt{2}} (p \uparrow \bar{n} \downarrow - p \downarrow \bar{n} \uparrow) \quad \pi^- = \frac{1}{\sqrt{2}} (n \uparrow \bar{p} \downarrow - n \downarrow \bar{p} \uparrow)$$

$$\pi^0 = \frac{1}{2} (p \uparrow \bar{p} \downarrow - p \downarrow \bar{p} \uparrow + n \uparrow \bar{n} \downarrow - n \downarrow \bar{n} \uparrow)$$

$$\omega_{+1} = \frac{1}{\sqrt{2}} (p \uparrow \bar{p} \uparrow - n \uparrow \bar{n} \uparrow) \quad \omega_{-1} = \frac{1}{\sqrt{2}} (p \downarrow \bar{p} \downarrow - n \downarrow \bar{n} \downarrow)$$

$$\omega_0 = \frac{1}{2} (p \uparrow \bar{p} \downarrow + p \downarrow \bar{p} \uparrow - n \uparrow \bar{n} \downarrow - n \downarrow \bar{n} \uparrow)$$

$$\eta_\omega = \frac{1}{2} (p \uparrow \bar{p} \downarrow - p \downarrow \bar{p} \uparrow - n \uparrow \bar{n} \downarrow + n \downarrow \bar{n} \uparrow)$$

$$\phi_{+1} = \lambda \uparrow \bar{\lambda} \uparrow \quad \phi_{-1} = \lambda \downarrow \bar{\lambda} \downarrow \quad \phi_0 = \frac{1}{\sqrt{2}} (\lambda \uparrow \bar{\lambda} \downarrow + \lambda \downarrow \bar{\lambda} \uparrow)$$

$$\eta_\phi = \frac{1}{\sqrt{2}} (\lambda \uparrow \bar{\lambda} \downarrow - \lambda \downarrow \bar{\lambda} \uparrow)$$

$$\omega = \sqrt{\frac{2}{3}} \omega_0 + \frac{1}{\sqrt{3}} \omega_8 \quad ; \quad \phi = \frac{1}{\sqrt{3}} \omega_0 - \sqrt{\frac{2}{3}} \omega_8 \quad \text{analogously for } \eta_\omega, \eta_\phi$$

NOTATION: QUARKS: $p = (I_3 = 1/2, Y = 1/3)$ $n = (I_3 = -1/2, Y = 1/3)$
 $\lambda = (I = 0, Y = -2/3)$ $\uparrow; \downarrow = (S_z = +1/2; -1/2)$

Table A.I.3

I^G	J^P	(MESON) ^{CHARGE} (HELICITY)	$ \underline{35} \rangle (L=1) L_z \rangle$ STATE
1^-	2^+	$\left\{ \begin{array}{l} A_2^i(h=0) = \frac{1}{\sqrt{6}} (\rho_{+1}^i\rangle -1 \rangle + \rho_{-1}^i\rangle +1 \rangle + 2 \rho_0^i\rangle 0 \rangle) \\ A_2^i(h=1) = \frac{1}{\sqrt{2}} (\rho_{+1}^i\rangle 0 \rangle + \rho_0^i\rangle +1 \rangle) \end{array} \right.$	
1^-	1^+	$\left\{ \begin{array}{l} A_1^i(h=0) = \frac{1}{\sqrt{2}} (\rho_{+1}^i\rangle -1 \rangle - \rho_{-1}^i\rangle +1 \rangle) \\ A_1^i(h=1) = \frac{1}{\sqrt{2}} (\rho_{+1}^i\rangle 0 \rangle - \rho_0^i\rangle +1 \rangle) \end{array} \right.$	
1^-	0^+	$\delta^i = \frac{1}{\sqrt{3}} (\rho_{+1}^i\rangle -1 \rangle + \rho_{-1}^i\rangle +1 \rangle - \rho_0^i\rangle 0 \rangle)$	
1^+	1^+	$\left\{ \begin{array}{l} B^i(h=0) = \pi^i\rangle 0 \rangle \\ B^i(h=1) = \pi^i\rangle +1 \rangle \end{array} \right.$	
0^+	2^+	$\left\{ \begin{array}{l} f(h=0) = \frac{1}{\sqrt{6}} (\omega_{+1}\rangle -1 \rangle + \omega_{-1}\rangle +1 \rangle + 2 \omega_0\rangle 0 \rangle) \\ f(h=1) = \frac{1}{\sqrt{2}} (\omega_{+1}\rangle 0 \rangle + \omega_0\rangle +1 \rangle) \end{array} \right.$	
0^+	1^+	$\left\{ \begin{array}{l} D(h=0) = \frac{1}{\sqrt{2}} (\omega_{+1}\rangle -1 \rangle - \omega_{-1}\rangle +1 \rangle) \\ D(h=1) = \frac{1}{\sqrt{2}} (\omega_{+1}\rangle 0 \rangle - \omega_0\rangle +1 \rangle) \end{array} \right.$	
0^+	0^+	$\epsilon = \frac{1}{\sqrt{3}} (\omega_{+1}\rangle -1 \rangle + \omega_{-1}\rangle +1 \rangle - \omega_0\rangle 0 \rangle)$	
0^-	1^+	$\left\{ \begin{array}{l} H_\omega(h=0) = \eta_\omega\rangle 0 \rangle \\ H_\omega(h=1) = \eta_\omega\rangle +1 \rangle \end{array} \right.$	

APPENDIX II

BARYON STATES IN $SU(6) \times O(3)$ AND ITS SUBGROUPS

We give here our notation and the explicit construction of the baryons as classified in $SU(6) \times O(3)$. The material is organized as follows:

Table A.II.1

- i) Notation for the non-strange $SU(6)$ states with charge +1;
- ii) Decomposition $SU(6) \supset SU(3) \times SU(2)$ for 56, 70, and 20 representations;
- iii) Same, with $SU(6) \supset SU(3) \times SU(3)$.

Table A.II.2

i) Three-particle states with definite symmetry under the group of permutations. They are: symmetric (S), mixed-symmetric (α and β) and antisymmetric (A). The states of the individual particles are defined by three sets of quantum numbers, x, y, and z. For example, the state (xyz) has

- the first particle in the state x
- the second particle in the state y
- the third particle in the state z.

ii) Same, with the states being described by two independent sets of quantum numbers: (x,y,z) and (a,b,c). For example, in the text (x,y,z) are the $SU(6)$ states of the three quarks in baryons (q_1, q_2, q_3) and (a,b,c), the spatial configuration of these quarks ($\vec{k}_1, \vec{k}_2, \vec{k}_3$).

Table A.II.3

Realization of the states in Table A.II.1 with the conventions of Table A.II.2. Notice that

- the 56 is symmetric (S)
- the two 70's have mixed symmetry (α and β)
- the 20 is completely antisymmetric (A).

Table A.II.4

Matrix elements of the axial charges for the different representations of the chiral $SU(3) \times SU(3)$ [generated by $Q^\alpha \pm Q_5^\alpha$]. They are given in terms of the $SU(3)$ Clebsch-Gordan coefficients defined at the bottom of the table. For example:

$$\begin{aligned} p \uparrow [I = \frac{1}{2}, I_3 = \frac{1}{2}, Y = 1] &\in (\underline{6}, \underline{3})_{\underline{8}}; \\ \Delta^+ \uparrow [I = \frac{3}{2}, I_3 = \frac{1}{2}, Y = 1] &\in (\underline{6}, \underline{3})_{\underline{10}}; \\ Q_5^3 [I = 1, I_3 = 0, Y = 0] & \end{aligned}$$

$$\begin{aligned} \langle p \uparrow | Q_5^3 | \Delta^+ \uparrow \rangle &= \frac{4}{3} \langle p | C^3 | \Delta^+ \rangle = \\ &= \frac{4}{3} \cdot \frac{\sqrt{3}}{\sqrt{2}} \left(\begin{array}{cc|c} \underline{8} & \underline{8} & \underline{10} \\ 1, 0 & \frac{1}{2}, 1 & \frac{3}{2}, 1 \end{array} \right) = \frac{4}{3\sqrt{2}} \end{aligned}$$

Table A.II.I

NOTATION FOR STATES WITH $Y=1, I_3=+\frac{1}{2}$

$SU(3) \times SU(2) \subset SU(6)$					$CHIRAL$
$SU(6)$	$SU(3)$		spin		$SU(3) \times SU(3) \subset SU(6)$
STATE	REPRES.	I	S	S_z	REPRESENTATION
<u>56</u>					
$p \uparrow$	<u>8</u>	$1/2$	$1/2$	$+1/2$	$(6, 3)$
$p \downarrow$				$-1/2$	$-(\bar{3}, 6)$
$\Delta \uparrow \uparrow$	<u>10</u>	$3/2$	$3/2$	$+3/2$	$(10, 1)$
$\Delta \uparrow$				$+1/2$	$(6, 3)$
$\Delta \downarrow$				$-1/2$	$(3, 6)$
$\Delta \downarrow \downarrow$				$-3/2$	$(1, 10)$
<u>70</u>					
$p_1 \uparrow$	<u>8</u>	$1/2$	$1/2$	$+1/2$	$\frac{1}{\sqrt{2}} [(6, 3) - (\bar{3}, \bar{3})]$
$p_1 \downarrow$				$-1/2$	$\frac{1}{\sqrt{2}} [-(3, 6) + (3, \bar{3})]$
$p_3 \uparrow \uparrow$	<u>8</u>	$1/2$	$3/2$	$+3/2$	$(8, 1)$
$p_3 \uparrow$				$+1/2$	$\frac{1}{\sqrt{2}} [(6, 3) + (\bar{3}, \bar{3})]$
$p_3 \downarrow$				$-1/2$	$\frac{1}{\sqrt{2}} [(3, 6) + (3, \bar{3})]$
$p_3 \downarrow \downarrow$				$-3/2$	$(1, 8)$
$\Delta_1 \uparrow$	<u>10</u>	$3/2$	$1/2$	$+1/2$	$(6, 3)$
$\Delta_1 \downarrow$				$-1/2$	$-(3, 6)$
<u>20</u>					
$p_{20} \uparrow$	<u>8</u>	$1/2$	$1/2$	$+1/2$	$(\bar{3}, 3)$
$p_{20} \downarrow$				$-1/2$	$-(3, \bar{3})$

Table A.II.2

a) 3-PARTICLE STATES WITH PERMUTATIONAL SYMMETRY

$$|xyz\rangle_S = \frac{1}{\sqrt{6}} (|xyz\rangle + |xzy\rangle + |yxz\rangle + |yzx\rangle + |zxy\rangle + |zyx\rangle)$$

$$|xyz\rangle_\alpha = \frac{1}{2\sqrt{3}} (2|zyx\rangle + 2|zxy\rangle - |xyz\rangle - |xzy\rangle - |yxz\rangle - |yzx\rangle)$$

$$|xyz\rangle_\beta = \frac{1}{2} (|yzx\rangle + |xzy\rangle - |yxz\rangle - |xyz\rangle)$$

$$|xyz\rangle_A = \frac{1}{\sqrt{6}} (-|xyz\rangle + |xzy\rangle - |yzx\rangle + |yxz\rangle - |zxy\rangle + |zyx\rangle)$$

b) COMBINATIONS OF TWO 3-PARTICLE STATES

$$|xyz, abc\rangle_S = \begin{cases} |xyz\rangle_S |abc\rangle_S \\ \frac{1}{\sqrt{2}} (|xyz\rangle_\alpha |abc\rangle_\alpha + |xyz\rangle_\beta |abc\rangle_\beta) \\ |xyz\rangle_A |abc\rangle_A \end{cases}$$

$$|xyz, abc\rangle_\alpha = \begin{cases} |xyz\rangle_S |abc\rangle_\alpha, |xyz\rangle_\alpha |abc\rangle_S \\ \frac{1}{\sqrt{2}} (|xyz\rangle_\alpha |abc\rangle_\alpha - |xyz\rangle_\beta |abc\rangle_\beta) \\ -|xyz\rangle_A |abc\rangle_\beta, -|xyz\rangle_\beta |abc\rangle_A \end{cases}$$

$$|xyz, abc\rangle_\beta = \begin{cases} |xyz\rangle_S |abc\rangle_\beta, |xyz\rangle_\beta |abc\rangle_\alpha \\ -\frac{1}{\sqrt{2}} (|xyz\rangle_\alpha |abc\rangle_\beta + |xyz\rangle_\beta |abc\rangle_\alpha) \\ |xyz\rangle_A |abc\rangle_\alpha, |xyz\rangle_\alpha |abc\rangle_A \end{cases}$$

$$|xyz, abc\rangle_A = \begin{cases} |xyz\rangle_S |abc\rangle_A, |xyz\rangle_A |abc\rangle_S \\ \frac{1}{\sqrt{2}} (|xyz\rangle_\beta |abc\rangle_\alpha - |xyz\rangle_\alpha |abc\rangle_\beta) \end{cases}$$

Table A.II.3

999 REALIZATION OF SOME OF THE $\gamma=0$ STATES

$$\underline{56}_s \left\{ \begin{array}{l} p \uparrow = \frac{1}{\sqrt{2}} (|ppn\rangle_\alpha | \uparrow \uparrow \downarrow \rangle_\alpha + |ppn\rangle_\beta | \uparrow \uparrow \downarrow \rangle_\beta) \\ \Delta \uparrow = |ppn\rangle \Delta \uparrow \quad \Delta \uparrow = |ppn\rangle_s | \uparrow \uparrow \downarrow \rangle_s \end{array} \right.$$

$$\left\{ \begin{array}{ll} p_3 \uparrow \uparrow = |ppn\rangle_\alpha | \uparrow \uparrow \uparrow \rangle_s & p_3 \downarrow \downarrow = |ppn\rangle_\alpha | \downarrow \downarrow \downarrow \rangle_s \\ p_3 \uparrow = |ppn\rangle_\alpha | \uparrow \uparrow \downarrow \rangle_s & p_3 \downarrow = |ppn\rangle_\alpha | \downarrow \downarrow \uparrow \rangle_s \end{array} \right.$$

$$\underline{70}_\alpha \left\{ \begin{array}{l} p_1 \uparrow = \frac{1}{\sqrt{2}} (|ppn\rangle_\alpha | \uparrow \uparrow \downarrow \rangle_\alpha - |ppn\rangle_\beta | \uparrow \uparrow \downarrow \rangle_\beta) \\ p_1 \downarrow = \frac{1}{\sqrt{2}} (|ppn\rangle_\alpha | \downarrow \downarrow \uparrow \rangle_\alpha - |ppn\rangle_\beta | \downarrow \downarrow \uparrow \rangle_\beta) \\ \Delta_1 \uparrow = |ppn\rangle_s | \uparrow \uparrow \downarrow \rangle_\alpha \quad \Delta_1 \downarrow = -|ppn\rangle_s | \downarrow \downarrow \uparrow \rangle_\alpha \end{array} \right.$$

$$\left\{ \begin{array}{ll} p_3 \uparrow \uparrow = |ppn\rangle_\beta | \uparrow \uparrow \uparrow \rangle_s & p_3 \downarrow \downarrow = |ppn\rangle_\beta | \downarrow \downarrow \downarrow \rangle_s \\ p_3 \uparrow = |ppn\rangle_\beta | \uparrow \uparrow \downarrow \rangle_s & p_3 \downarrow = |ppn\rangle_\beta | \downarrow \downarrow \uparrow \rangle_s \end{array} \right.$$

$$\underline{70}_\beta \left\{ \begin{array}{l} p_1 \uparrow = \frac{1}{\sqrt{2}} (|ppn\rangle_\beta | \uparrow \uparrow \downarrow \rangle_\alpha + |ppn\rangle_\alpha | \uparrow \uparrow \downarrow \rangle_\beta) \\ p_1 \downarrow = \frac{1}{\sqrt{2}} (|ppn\rangle_\beta | \downarrow \downarrow \uparrow \rangle_\alpha + |ppn\rangle_\alpha | \downarrow \downarrow \uparrow \rangle_\alpha) \\ \Delta_1 \uparrow = |ppn\rangle_s | \uparrow \uparrow \downarrow \rangle_\beta \quad \Delta_1 \downarrow = -|ppn\rangle_s | \downarrow \downarrow \uparrow \rangle_\beta \end{array} \right.$$

$$\underline{20}_A \left\{ \begin{array}{l} p_{20} \uparrow = \frac{1}{\sqrt{2}} (|ppn\rangle_\alpha | \uparrow \uparrow \downarrow \rangle_\beta - |ppn\rangle_\beta | \uparrow \uparrow \downarrow \rangle_\alpha) \\ p_{20} \downarrow = \frac{1}{\sqrt{2}} (|ppn\rangle_\alpha | \downarrow \downarrow \uparrow \rangle_\beta - |ppn\rangle_\beta | \downarrow \downarrow \uparrow \rangle_\alpha) \end{array} \right.$$

Table A.II.4

SU(3) MATRIX ELEMENTS OF THE AXIAL CHARGES
FOR THE REPRESENTATIONS OF THE CHIRAL SU(3) x SU(3)

$$\langle (\underline{6}, \underline{3})_8 | Q_5 | (\underline{6}, \underline{3})_8 \rangle = D + \frac{2}{3} F$$

$$\langle (\underline{\bar{3}}, \underline{3})_8 | Q_5 | (\underline{\bar{3}}, \underline{3})_8 \rangle = -D$$

$$\langle (\underline{8}, \underline{1}) | Q_5 | (\underline{8}, \underline{1}) \rangle = F$$

$$\langle (\underline{10}, \underline{1}) | Q_5 | (\underline{10}, \underline{1}) \rangle = E$$

$$\langle (\underline{6}, \underline{3})_{10} | Q_5 | (\underline{6}, \underline{3})_{10} \rangle = \frac{1}{3} E$$

$$\langle (\underline{6}, \underline{3})_{10} | Q_5 | (\underline{6}, \underline{3})_8 \rangle = \frac{4}{3} C$$

$$\langle (\underline{\bar{3}}, \underline{3})_8 | Q_5 | (\underline{\bar{3}}, \underline{3})_1 \rangle = -\frac{\sqrt{2}}{\sqrt{3}}$$

$$\langle (\underline{R}, \underline{S}) | Q_5 | (\underline{R}, \underline{S}) \rangle = -\langle (\underline{S}, \underline{R}) | Q_5 | (\underline{S}, \underline{R}) \rangle$$

$(\underline{6}, \underline{3})_8$ MEANS: THE OCTET PART OF THE $(\underline{6}, \underline{3})$, ETC.

THE D's, F's, E's, C's ARE SU(3) CLEBSCH-GORDAN COEFFICIENTS, DEFINED AS FOLLOWS:

$$F = \frac{1}{\sqrt{3}} (\underline{8} \ \underline{8} | \underline{8}_a)$$

$$D = \sqrt{\frac{5}{3}} (\underline{8} \ \underline{8} | \underline{8}_s)$$

$$E = \sqrt{6} (\underline{10} \ \underline{8} | \underline{10})$$

$$C = \sqrt{\frac{3}{2}} (\underline{8} \ \underline{8} | \underline{10})$$

APPENDIX III

DERIVATION OF THE MAGNETIC MOMENT FOR NEUTRAL MESONS

In this Appendix we outline the derivation of Eq. (IV.16). The calculations are merely a transposition of the method used, for example, in Ref. 8.

$$\begin{aligned}
 \langle \tilde{\pi}^0 | \mu_y | \tilde{\omega} \rangle &= \langle \tilde{\pi}^0 | \int d^3\vec{x} (z J_x^{EM}(\vec{x}) - x J_z^{EM}(\vec{x})) | \tilde{\omega} \rangle \\
 &\xrightarrow{P_z \rightarrow \infty} - \int d^3x \langle \tilde{\pi}^0 | x J_0^{EM}(\vec{x}) | \tilde{\omega} \rangle = \\
 &= i \frac{\partial}{\partial k_x} \langle \tilde{\pi}^0, k_x | J_0^{EM}(k_x) | \tilde{\omega} \rangle \Big|_{k_x=0} = \langle \tilde{\pi}^0 | \left[i \frac{\partial}{\partial k_x} J_0^{EM}(k_x) - \frac{J_y Q^{EM}}{P_z} \right] | \tilde{\omega} \rangle \Big|_{k_x=0} = \\
 &= i \langle \pi^0 | e^{-i\theta Z} \frac{\partial}{\partial k_x} J_0^{EM}(k_x) e^{i\theta Z} | \omega \rangle \Big|_{k_x=0} = i \langle \pi^0 | \bar{e}^{-i\theta Z} \frac{\partial}{\partial k_x} e^{i\theta Z} J_0^{EM} | \omega \rangle \\
 &= \langle \pi^0 | \left\{ i \frac{\partial}{\partial k_x} + \frac{(\bar{\sigma}_y + \bar{\sigma}_y)(\omega^2 - \omega m - k_x^2) + (\bar{\sigma}_x + \bar{\sigma}_x) k_x k_y}{2\omega^2(\omega + m)} \right\} J_0^{EM} | \omega \rangle,
 \end{aligned}$$

which gives Eq. (IV.16) as both states have $L = 0$.

Remark: In taking the derivative with respect to k_x , one must not use Eq. (IV.4) for θZ ,

$$\theta Z = \frac{[(\vec{\sigma} - \vec{\bar{\sigma}}) \times \vec{k}]}{2k} \arctan \frac{k}{m}$$

but rather one should take the original expression, where Eq. (IV.4) has been derived from,

$$\theta Z = \frac{1}{2} \arctan \frac{[\vec{\sigma} \times \vec{k}]_z}{m} - \frac{1}{2} \arctan \frac{[\vec{\bar{\sigma}} \times \vec{k}]_z}{m}.$$

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ACKNOWLEDGEMENTS

This work has been performed in the stimulating atmosphere of the Département de Physique Théorique and I am indebted to my colleagues there. Special thanks are due to: Professor H. Ruegg for his continuous support ever since I joined his group, and especially for his valuable suggestions and comments during the development of this work; Professors J.-M. Jauch and D. Amati for participating as members of the thesis jury; S. McGarry for careful typing and for her friendly and efficient help in the redaction of the manuscript; F. Buccella for his stimulating collaboration on a part of this work; Professors A.H. Zimmerman and G. Costa for introducing me into the study of high-energy physics.

